AMERICAN JOURNAL of PHYSICS

A Journal Devoted to the Instructional and Cultural Aspects of Physical Science

VOLUME 15, NUMBER 1 JANUARY–FEBRUARY, 1947

The Physics of Rockets

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I. PRINCIPLES OF ROCKET PROPULSION

ALTHOUGH the Chinese are credited with the use of gunpowder rockets as early as several centuries B.C., and Hero of Alexandria invented a steam jet propulsion device about 100 B.C., most of the serious effort to develop rockets has occurred in the last three decades. Goddard[1] in America made a complete study of rocket performance in 1914. The German all-out rocket program began in 1935 and culminated in the V-2, which was first fired in September 1944. Since 1938, intensive rocket research has been carried out by a number of American agencies; a basic theoretical contribution was made by Malina[2] in 1940.

The present paper will concern itself only with that type of jet-propulsion device designated as a "pure" rocket, that is, a thrust producer which does not make use of the surrounding atmosphere. This restriction excludes propulsive duct devices such as the "turbojet" engine used in jet-propelled airplanes of the P-80 type. No attempt will be made to discuss the aerodynamics of bodies moving with supersonic speeds, the electronic problems of rocket-missile guidance and control, the measurement of physical quantities in the upper atmosphere, or the properties of artillery rockets. Even omitting these interesting fields, the science of rocketry embraces many phases of physics and chemistry, as will appear in later sections.

The Chinese symbol for "rocket"—literally "fire arrow" (reading from top to bottom).

1. Physical Nature of Reaction Propulsion

A rocket is a rigid container for matter and energy so arranged that a portion of the matter can absorb the energy in kinetic form and subsequently be ejected in a specified direction. The matter, originally stationary relative to the container, is emitted usually as a continuous fluid with an "exhaust" velocity v and a mass flow rate $\dot{m}[=dm/dt]$, thus undergoing a time-rate of change of momentum $\dot{m}v$. This rate of change of momentum is transmitted to the residual solid portion, of instantaneous total mass m, of the rocket as a reaction force, or "thrust,"

$$F = \dot{m}v, \tag{1}$$

where F and v are in opposite senses because $\dot{m}$ is negative. Equation (1) holds if the mass is ex-

[1] R. H. Goddard, "A method of reaching extreme altitudes," *Smithsonian Misc. Collections* **71**, No. 2 (1919).

[2] F. J. Malina, "Characteristics of the rocket motor unit based on the theory of perfect gases," *J. Frank. Inst.* **230**, 433–454 (1940).

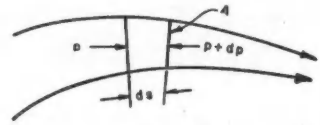

FIG. 1. Adiabatic expansion of a compressible fluid.

hausted into a vacuum. Thus the basic mechanics of a rocket appear disarmingly simple.

An ideal rocket is a device for producing maximum rate of change of momentum (and hence thrust force) by means of a minimum expenditure of mass. A brief calculation will show that, for a constant thrust, the rate at which kinetic energy is supplied to the expended mass varies inversely as the mass flow rate. Since an effective rocket should spend mass frugally, it must squander energy prodigally. This high rate of evolution of energy implies that the ejected matter is subjected to high temperatures and consequently is in the gaseous state. In the discussion that follows, the exhaust products will be assumed to follow the laws of ideal gases. The goal of a rocket "motor," then, will be to convert random thermal energy of a gas into an ordered state in which the motions of all the molecules have been collimated in a specific direction. In this ideal condition their macroscopic momentum will be a maximum and their temperature and pressure essentially zero. Since this would require expansion into a vacuum, a rocket operating in the earth's atmosphere cannot achieve perfect efficiency. The expansion process must stop when the pressure of the emerging jet equals that of the surrounding atmosphere. Consequently the efficiency of the motor has limitations.

2. Criterions for Rating Rocket Performance

The rocket is unusual in the field of propulsion devices in that its thrust is independent of its velocity and does not require the presence of surrounding matter. This contrasts with the airplane power plant, for example, the thrust of which decreases with increasing relative velocity and decreasing density of the atmosphere. Conventional motors normally propel their loads at a constant speed; rocket motors usually are accelerating a free body of rapidly decreasing mass. The goal of a conventional motor is to exert a force through a distance; that of a rocket, to exert a force during a time interval, that is, to achieve a given terminal velocity. Therefore, impulse (or momentum change) is a more significant parameter in rating rockets than energy dissipated, and the thrust per unit weight-rate of flow—called the "specific impulse,"

$$I_{sp} = Ft/m_p g = F/\dot{m}_{av} g, \tag{2}$$

—is a more useful measure of performance than is the power generated. Moreover, it is found that the total mass of propellant m_p necessary is the same for the same total impulse, no matter whether the impulse is delivered as a large thrust for a short time or a small thrust for a long time.

The propulsive power developed by a rocket is proportional to its speed. For example, the German V-2 rocket at its maximum speed of 5000 ft/sec develops over half a million horsepower, whereas the power immediately after take-off is relatively low.

The reciprocal of I_{sp} is the "specific propellant consumption" $\dot{w}_{sp}[=1/I_{sp}]$. The product $I_{sp}g$ is the "effective exhaust velocity" c, which is essentially the velocity v appearing in Eq. (1). The velocities c and v differ in that c is defined by the equation

$$c = gI_{sp} \tag{3}$$

and may have a different value from the true velocity of efflux v, owing to inefficiencies such as the back pressure of the atmosphere on the jet. Although I_{sp} is primarily a measure of propellant performance, its value is affected by the geometric design of the rocket, the combustion pressure and the external atmospheric pressure. This must be remembered when comparing propellants.

The impulse-weight ratio,

$$\frac{Ft}{(m_p+m_0)g} = \frac{\text{impulse}}{\text{total weight}}, \tag{4}$$

is a measure of the performance of the rocket, in which the nonexpendable mass m_0 is taken into account. It indicates the excellence of the over-all design of container-plus-propellant as a unit. Certain rocket propellants whose specific impulses are higher than those of others may lose their advantage when compared on the basis of impulse-weight ratio, owing to their low density.

Dynamics of Rocket Jets

3. Basic Thermodynamic Relations

In this and the following sections quantitative relations will be presented for the velocity of the gases issuing from the exhaust nozzle of a rocket, as well as the magnitude of the thrust therefrom. For the most part the theory will be developed from first principles for the sake of clarity and continuity, even though certain portions are well documented elsewhere.

Up to this point only the relation between thrust and *mechanical* energy has been considered. Assuming that a rocket propellant with a certain heat of combustion is to be used, one must now investigate the *thermodynamic* process of converting this heat into useful thrust-producing mechanical energy.

Let dq be the additional heat introduced into a unit mass of gas; dE, the change in the internal energy[3] of the gas; dV, the change in volume of the gas due to this heating; and p, the pressure under which the process is carried out. Then pdV is the work done by the gas in expansion. According to the first law of thermodynamics,

$$dq = dE + pdV. \tag{5}$$

In general, the internal energy E is a function of the gas temperature T and, if Van der Waals forces exist, of the specific volume V_s also.

The specific heat at constant volume, $c_v[=(\partial q/\partial T)_v]$, is, by Eq. (5),

$$c_v = (\partial E/\partial T)_v = \text{const.} \tag{6}$$

Hence c_v is equal to the temperature-rate of change of internal energy at constant volume. By transforming Eq. (5) into a new form one may arrive at an expression for the specific heat at constant pressure, c_p. Thus

$$dq = d(E+pV) - Vdp = dH - Vdp, \tag{7}$$

where the quantity $H[=E+pV]$ is the enthalpy, or heat content, of the fluid. From Eq. (7),

$$c_p = (\partial q/\partial T)_p = (\partial H/\partial T)_p = \text{const.} \tag{8}$$

Hence c_p is equal to the rate of change of enthalphy at constant pressure.

We shall now relate the "mechanical" velocity v of a gas to its thermodynamic attributes of pressure, temperature and density. Mechanical velocity is used here in the sense of bulk, or gross, velocity, as distinguished from random thermal motion of the molecules. A form of the Bernoulli equation applicable to compressible fluids will be employed. Consider a stream of gas (Fig. 1) in steady flow, and compute the acceleration of the gas element located between two closely neighboring cross sections, each of area A. The fact that the stream is in steady flow does not, of course, mean that individual elements of gas are unaccelerated.

Applying Newton's second law of motion to the gas element of thickness ds, one gets

$$\rho A ds dv/dt = pA - (p+dp)A = -Adp, \tag{9}$$

where ρ is the density of the gas. In steady flow the velocity of the gas elements passing by any fixed point on the stream will not change with time. However, it will change with position along the stream. Thus,

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial s}\cdot\frac{ds}{dt} = 0 + v\frac{dv}{ds}. \tag{10}$$

Combining Eqs. (10) and (9), we get the Bernoulli equation,

$$-dp = \rho v dv. \tag{11}$$

The negative sign indicates that velocity increases as pressure decreases. Since we are considering a unit mass, $\rho = 1/V_s$, where V_s is the specific volume. If we use this substitution in Eq. (11), which is essentially Newton's second law, and combine the result with the first law of thermodynamics in the form of Eq. (7), we obtain

$$dq = dH + vdv = d(H + \tfrac{1}{2}v^2). \tag{12}$$

Since H is the enthalpy per unit mass, the term $\frac{1}{2}v^2$ is equivalent to kinetic energy per unit mass. Therefore, the thermodynamic quantities have been combined with the gross mechanical velocity in a useful and fundamental relation. If we assume further that the stream shown in Fig. 1 is undergoing an adiabatic process—for example, expansion through a rocket nozzle—then $dq = 0$, and Eq. (12) becomes, upon integration,

$$H + \tfrac{1}{2}v^2 = \text{const.} \tag{13}$$

Equation (13) states that the *sum of the enthalpy and the kinetic energy per unit mass of a gas is constant in steady, adiabatic flow.* Fortunately, this is the case that is applicable to most rockets.

4. Adiabatic Flow

In order to derive quantitative relations describing adiabatic flow ($dq = 0$), it is necessary to restrict the properties of the compressible fluid to those of an ideal gas. By definition, the internal energy E and enthalpy H of an ideal gas are functions of temperature only. Thus, if Eqs. (5) and (6), or (7) and (8) are combined, the first law of thermodynamics takes the forms

$$c_v dT + pdV = 0, \tag{14a}$$

$$c_p dT - Vdp = 0. \tag{14b}$$

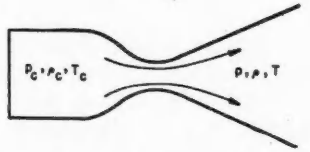

FIG. 2. Thermodynamic parameters in a rocket chamber and in an arbitrary section of the nozzle.

[3] See, for instance, Millikan, Roller and Watson, *Mechanics, molecular physics, heat and sound* (Ginn, 1937), p. 264.

Eliminating dT from Eqs. (14a) and (14b), and introducing the ratio of the specific heats $\gamma[=c_p/c_v]$, we have

$$\gamma(dV/V)+(dp/p)=0. \tag{15}$$

Up to this point there have been no restrictions on c_p and c_v, which may be either variable or constant. If we now assume that their ratio γ is constant, Eq. (15) is integrable to the well-known adiabatic relation,

$$pV^\gamma=\text{const.} \tag{16}$$

The definition of an ideal gas is completed by requiring not only that E be a function of T only, but also that the gas be subject to the equation of state for unit mass,

$$pV_s=(R/M)T=R_sT, \tag{17}$$

where R is the universal gas constant [=1543 ft lb/mole deg F], M is the effective molecular weight[4] of the gas and R_s is a specific gas constant.

By taking the difference of Eqs. (14a) and (14b) and combining this with Eq. (17) we obtain, after rearrangement

$$c_p-c_v=R_s. \tag{18}$$

Combining this result with the definition $\gamma=c_p/c_v$ leads to a pair of relations that will later be useful:

$$c_p=\frac{\gamma}{\gamma-1}R_s, \tag{19a}$$

and

$$c_v=\frac{1}{\gamma-1}R_s. \tag{19b}$$

By manipulation of Eqs. (16) and (17), one may express either pressure, temperature, density or specific volume in terms of one of the remaining two variables. Furthermore, the ratio of, say, temperatures at any two points in an adiabatic cycle may be expressed as a simple power of the ratio of the values of any other variable at the same points. These relations are particularly useful in evaluating a parameter at any point in the rocket nozzle in terms of the value of the same quantity in the combustion chamber, at which point it is more readily measured. For instance,

$$T/T_c=(p/p_c)^{[(\gamma-1)/\gamma]}, \tag{20}$$

where the subscript c refers to the conditions in the combustion chamber. Since $(\gamma-1)/\gamma \doteq \frac{1}{5}$ for a rocket exhaust, Eq. (20) shows that a relatively large change in pressure gives only a small change in temperature as the gases expand adiabatically through the exhaust nozzle. This equation illustrates also the importance of the parameter γ in the dynamics of compressible fluids.

5. Velocity Obtainable by Adiabatic Expansion

We have now developed the necessary relations for expressing the exhaust velocity of the gases in a rocket nozzle explicitly in terms of temperature or pressure. From the definition of c_p in Eq. (8) and the fact that c_p is constant in an ideal gas, we have by integration

$$H=c_pT+H_0, \tag{21}$$

where H_0 is a constant of integration.

Referring to Fig. 2, let us denote the pressure and temperature in the combustion chamber of a rocket by p_c and T_c, and those at any downstream section of the nozzle by p and T. Since the velocity of the gas in the combustion chamber is small, we have from Eqs. (13) and (21),

$$\tfrac{1}{2}v^2-0=c_p(T_c-T). \tag{22}$$

But c_p may be expressed in terms of general constants by Eq. (19a); thus

$$v^2=[2\gamma/(\gamma-1)]R_sT_c(1-T/T_c). \tag{23}$$

Using Eqs. (17) and (20) we can replace temperatures by pressures and densities, which are easier to determine, and finally arrive at an expression for the exhaust velocity v, corresponding to an exhaust pressure p, namely,

$$v=\left\{2\frac{\gamma}{\gamma-1}\cdot\frac{p_c}{\rho_c}\left[1-\left(\frac{p}{p_c}\right)^{(\gamma-1)/\gamma}\right]\right\}^{\frac{1}{2}}. \tag{24}$$

A number of interesting conclusions may be drawn from Eq. (24). The factor involving p/p_c increases toward unity as p/p_c approaches zero. Thus, by expanding the gas to lower pressure, the exhaust velocity may be increased. The maximum velocity v_{max} is obtained if $p=0$, that is, if the gas expands into a vacuum. In this case, all the thermal energy of the gas is converted into kinetic energy. From Eq. (24) the expression for v_{max} is

$$v_{max}=\left(2\frac{\gamma}{\gamma-1}\,\frac{p_c}{\rho_c}\right)^{\frac{1}{2}}. \tag{25}$$

For a given application a rocket is designed for a definite ratio p/p_c of the pressures in the atmosphere and in the chamber. The atmospheric pressure p may vary from 14.7 lb/in.² to zero, depending on the rocket's flight path. A typical value of p/p_c for liquid propellants is 14.7/300. For a given value of p/p_c, Eq. (24) shows that v increases as γ decreases. Therefore it is advantageous to have a gas for which γ is small, although in practice little control can be exercised over this parameter.

From Eq. (17) we see that $p_c/\rho_c=(R/M)T_c$. This indicates that the chamber temperature T_c should be high and the molecular weight M of the products of combustion should be low to secure a high exhaust velocity v. That M should be low may be seen in another way by using the principle of equipartition of energy.[5] Consider two combustion chambers at the same pressure p_c and temperature

[4] The quantity M must be expressed in mass (not weight) units.

[5] See, for example, reference 3, p. 210.

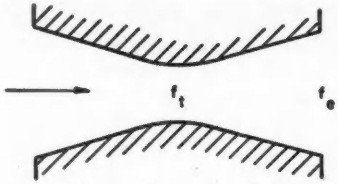

FIG. 3. The de Laval, or converging-diverging supersonic nozzle, showing throat of area f_t and exit section of area f_e.

T_c containing two species of ideal gas molecule, one of mass M, the other of mass $100M$. If the average kinetic energy of the lighter molecules is $\frac{1}{2}Mv^2$, then that of the heavier is, according to the equipartition theory, the same, namely, $\frac{1}{2}(100M)(v/10)^2$. When the gas is discharged into a vacuum, the average momentum of the lighter molecules will be Mv, and that of the heavier, $(100M)(v/10)$, or $10Mv$. However, for *equal mass flow rates* 100 of the lighter molecules will be discharged for each heavy one, so that the net momentum change and hence thrust from the chamber containing the lighter species will be ten times that of the chamber containing the heavier molecules, even though they are at the same temperature and pressure and have the same discharge rates. This fact is important in determining the choice of rocket propellants, and favors those that contain a high percentage of hydrogen.

If the two rockets in the foregoing example had equal *thrusts* rather than equal mass flow rates, then the mass flow rate of the lighter molecules would be one-tenth that of the heavier, but the power supplied to the lighter molecules would be ten times that supplied to the heavier. This is consistent with the statement made in SEC. 1.

Flow Through Nozzles

6. Discharge Velocity and the Velocity of Sound

The factor $p_c/\rho_c[=RT_c]$ in Eq. (24) for the velocity is really a measure of the total thermal energy stored in the gas, and has the dimensions of energy per unit mass. For ideal gases this is simply the kinetic energy of molecules moving with a certain mean speed. Since a wavelike disturbance is propagated through a gas with a velocity that depends upon this same mean molecular speed of thermal agitation, it might be intuitively expected that the velocity of sound waves in a gas and the velocity acquired by the molecules of the same gas in free adiabatic expansion would be simply related. A good measure of the random velocity of molecular motion is the so-called velocity of sound[6] a, defined by the equation

$$a^2=\mathrm{d}p/\mathrm{d}\rho. \qquad (26)$$

[6] See, for example, reference 3, pp. 363ff.

For adiabatic processes, Eqs. (15) and (17) reduce Eq. (26) to

$$a^2=\gamma p/\rho=\gamma R_s T. \qquad (27)$$

Hence, if we denote the velocity of sound in the combustion chamber by a_c, then Eq. (25) for the maximum exhaust velocity becomes

$$v_{\max}=a_c\left(\frac{2}{\gamma-1}\right)^{\frac{1}{2}}, \qquad (28a)$$

and Eq. (24) for the exhaust velocity at any pressure p becomes

$$v=a_c\left\{\frac{2}{\gamma-1}\left[1-\left(\frac{p}{p_c}\right)^{(\gamma-1)/\gamma}\right]\right\}^{\frac{1}{2}}. \qquad (28b)$$

For many rocket propellants, $\gamma=1.25$, so that $v_{\max}=2.828a_c$. For a typical case of expansion from 500 to 15 lb/in.^{2}, the local velocity of sound at the nozzle exit is $\frac{1}{2}a_c$, and the ratio of exit exhaust velocity to the local velocity of sound v/a—called the "Mach number" M—has a value of 5 or 6. Such flow is designated as *supersonic* flow and can be achieved only with a properly shaped exhaust nozzle.

7. The de Laval Exhaust Nozzle

It is necessary in designing rockets to express the thrust $[=\dot{m}v]$ in terms of known properties of the combustion products and pressures. This has already been done for the exhaust velocity in Eq. (24). It remains to be shown how the mass flow rate $\dot{m}$ can be expressed in the same terms, and what effect the back pressure of the atmosphere has on thrust, before the latter can be calculated explicitly in terms of dimensions and pressures.

The geometry of the orifice through which the compressible gases escape is important in determining both mass flow rate and thrust. It is not evident *a priori* what the shape of this "nozzle" should be. For example, the velocity of an incompressible fluid such as water flowing through a converging-diverging venturi tube first increases and then decreases, with a maximum at the smallest cross section. The mass flow rate is proportional to the over-all pressure difference. On the other hand, a compressible fluid undergoing adiabatic expansion through a

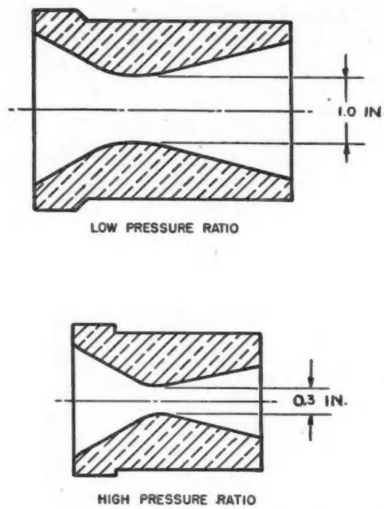

FIG. 4. Two typical nozzle contours. The upper one is designed to produce a thrust of 300 lb by expanding gases from a pressure 300 lb/in.[2] (liquid propellant) and the lower one the same thrust by expansion from 2000 lb/in.[2] (solid propellant).

similar venturi will behave in the same manner only so long as the velocity at all points is less than the local velocity of sound. As soon as sonic velocity is reached, which occurs first at the narrowest cross section, or "throat," the behavior changes entirely. The mass flow rate (but not the velocity) is now unaffected by any changes in pressure downstream from the throat. This effect is sometimes called "nozzling." Moreover, the gas velocity downstream of the throat will *increase* (becoming supersonic) to a value determined by the pressure at the nozzle exit. The pressure difference necessary to cause sonic flow is called the *critical* pressure difference. The pressure difference in all rocket chambers is well above this initial value. Quantitative relations to verify these statements will now be derived.

Since the mass flow rate $\dot{m}$ at every cross section of a nozzle is constant, we have the continuity condition

$$\dot{m} = f\rho v = \text{const.}, \tag{29}$$

where f is the area of any cross section, and ρ and v are the density and velocity at that cross section. It is possible to express both ρ and v in terms of the ratio p/p_c, resulting in a relation between the pressure p at any cross section and the area f of the same cross section for a specified mass flow rate $\dot{m}$. The value of ρ may be expressed in terms of pressure by an equation similar to Eq. (20), namely,

$$\rho = \rho_c(p/p_c)^{1/\gamma}. \tag{30}$$

Putting this value of ρ, and that of v from Eq. (24), in Eq. (29), and solving for f, we obtain

$$f = \frac{\dot{m}}{\rho_c}\left(\frac{p}{p_c}\right)^{-(1/\gamma)}\left\{\frac{2\gamma}{\gamma-1}\cdot\frac{p_c}{\rho_c} \times\left[1-(p/p_c)^{(\gamma-1)/\gamma}\right]\right\}^{-\frac{1}{2}}. \tag{31}$$

If the area f is calculated from Eq. (31) for a series of steadily decreasing values of p/p_c, it is found that f has a minimum value.[7] This indicates that in order for p to decrease (and hence v to *increase*) steadily, the nozzle should be shaped as in Fig. 3. A nozzle having this contour is called a de Laval nozzle, after the Swedish engineer Carl G. F. de Laval, who first used it to obtain supersonic gas velocities.

The fact that a supersonic nozzle must have the convergent-divergent contour may be shown in another way. The equation of continuity (29) written in differential form is

$$\frac{df}{f}+\frac{d\rho}{\rho}+\frac{dv}{v}=0. \tag{32}$$

The differential form of the Bernoulli law, Eq. (11), may be

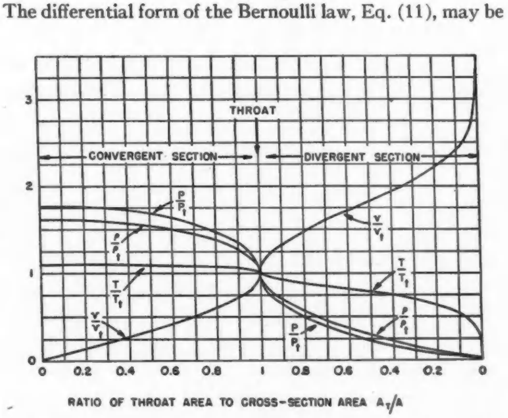

FIG. 5. Dimensionless graph of supersonic nozzle velocity and thermodynamic parameters, referred to values at the throat, as functions of the relative area. The gas flow is from left to right.

[7] Roberts, *Heat and thermodynamics* (Blackie, 1940), ed. 3, p. 293.

rewritten with the help of Eq. (26) as follows:

$$\frac{dp}{\rho}=\frac{dp}{d\rho}\frac{d\rho}{\rho}=a^2\frac{d\rho}{\rho}=-v dv. \tag{33}$$

By substituting this expression for $d\rho/\rho$ in Eq. (32), remembering the definition of Mach number $M[=v/a]$, one gets

$$\frac{df}{f}=-\frac{dv}{v}(1-M^2). \tag{34}$$

Equation (34) shows that if the velocity increases continuously along the nozzle—that is, if dv is always positive —then

when v is subsonic,	$M<1$,	$df/f<0$,	f is decreasing;
when v is sonic,	$M=1$,	$df/f=0$,	f is minimum;
when v is supersonic,	$M>1$,	$df/f>0$,	f is increasing.

The point of minimum cross-sectional area f_t is called the *throat*. The value of the pressure at the throat p_t may be found by differentiating Eq. (31) with respect to p/p_c and setting the derivative equal to zero. In this way one obtains

$$p_t/p_c=\left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)}. \tag{35}$$

Combining Eqs. (35) and (20), one finds for the temperature at the throat,

$$T_t/T_c=2/(\gamma+1). \tag{36}$$

The pressure ratio given by Eq. (35) is called the *critical* pressure ratio. In regions where the value of the pressure ratio is less than this initial value, the nozzle is convergent throughout; elsewhere it is convergent-divergent. By substituting the value of the critical pressure ratio from Eq. (35) in Eq. (24), and using Eqs. (20) and (27), we find that the velocity at the throat v_t is

$$v_t=\left(\frac{2\gamma}{\gamma+1}\cdot\frac{p_c}{\rho_c}\right)^{\frac{1}{2}}=\left(\frac{2\gamma R_s T_c}{\gamma+1}\right)^{\frac{1}{2}}$$
$$=(\gamma R_s T_t)^{\frac{1}{2}}=a_t; \tag{37}$$

this is just the *velocity of sound under the conditions which prevail in the throat.*

The area of the throat f_t necessary for a given mass flow rate $\dot{m}$ with a given pressure ratio can be calculated by substituting in Eq. (31) the value of p_t/p_c from Eq. (35), providing the pressure ratio exceeds the critical value. Thus, the value of pressure p downstream of the throat has no effect on the mass flow rate. This seemingly anomalous phenomenon is *not* predicted by the application of the Bernoulli equation, but occurs in contradiction to it.[8] It is a consequence of the finite velocity of sound and the fact that a reduction in pressure beyond the throat cannot be propagated upstream through the fluid, because the latter has a supersonic velocity. This explanation was first pointed out by Osborne Reynolds.[9]

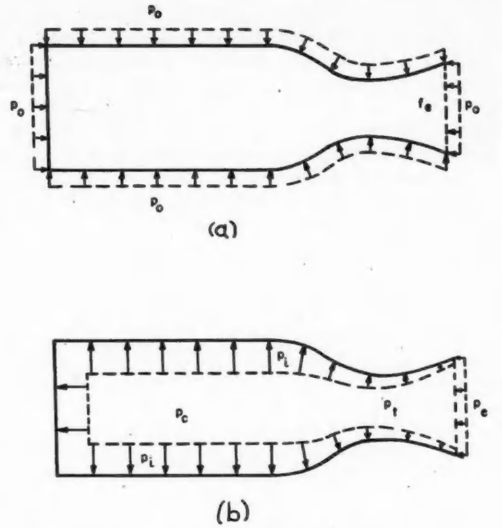

FIG. 6. Pressure forces acting on a rocket motor to produce the thrust: (*a*) the external forces due to atmospheric pressure; (*b*) the internal forces due to the combustion gases.

It is interesting to note that although Eqs. (31) and (34) for the area f indicate that a minimum in the area is necessary if the gas velocity v is to increase continuously, and although Eq. (31) relates pressure p and area f uniquely, it gives no other information as to the actual geometric shape of the nozzle. This shape is in fact not unique, and other less fundamental considerations such as weight and heat transfer determine the precise contour. Two characteristic nozzle contours are shown in Fig. 4 for (a) a liquid propellant of low combustion pressure and (b) a solid propellant of high combustion pressure.

A brief qualtitative résumé of the behavior of the de Laval nozzle at various pressure ratios is of interest. If one maintains a fixed chamber pressure but gradually reduces

[8] O'Brien and Hickox, *Applied fluid mechanics* (McGraw-Hill, 1937), p. 43.

[9] O. Reynolds, *Phil. Mag.* (5) **21**, 185 (1886).

the exit pressure to a value only a little below this chamber pressure, for a given nozzle the flow is first all subsonic and the nozzle is actually a venturi tube. The velocity first increases, but decreases again beyond the throat section. The velocity at the throat section will increase with continued reduction of the exit pressure, and hence the mass flow rate $\dot{m}$ through the nozzle will also increase. After sonic velocity is reached at the throat section, further reduction in exit pressure will not increase $\dot{m}$, which will remain constant because the sonic throat velocity remains constant. However, the velocity of exit will be increased by decreasing the pressure at the nozzle exit, with accompanying complicated shock-wave[10] patterns, until the pressure just outside the nozzle has become that given by the area ratio[11] of the nozzle. Further reduction in "atmospheric pressure" will not increase the exit velocity, because the additional expansion of the gas takes place outside the nozzle. This last situation may result in an increase in the thrust F due to the pressure effect, as will be shown in SEC 9. Figure 5 shows how the parameters p, ρ, v and T vary throughout the nozzle relative to their values in the throat section.

8. Mass Flow Through the Nozzle

The mass flow through the nozzle can be expressed in terms of chamber conditions and the area of the throat by combining the continuity equation and previously derived relations in the following manner. From Eq. (29),

$$\dot{m}=f_t\rho_t v_t, \tag{38}$$

where the subscript t refers to throat conditions.

The sonic velocity at the throat v_t is given by Eq. (37), and the density of the throat ρ_t may be expressed in terms of p_t and T_t by means of Eq. (17). But p_t and T_t are known in terms of chamber conditions from Eqs. (35) and (36). Making all these substitutions in Eq. (38) results in

$$\dot{m}=\Gamma'\frac{f_t p_c}{(\gamma R_e T_c)^{\frac{1}{2}}}=\frac{\Gamma' f_t p_c}{a_c}, \tag{39}$$

where $\Gamma'(\gamma)$ is a constant defined by the equation

$$\Gamma'\equiv\gamma\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/2(\gamma-1)}. \tag{40}$$

It is apparent from Eq. (39) that $\dot{m}$ is independent of quantities downstream from the throat. The velocity a_c is not directly measurable, but it will be shown in SEC. 10 that a_c cancels out in practical design calculations of $\dot{m}$. The mass flow rate $\dot{m}$ is used in calculating the thrust F of a rocket.

[10] Durand, *Aerodynamic theory* (Springer, Berlin, 1934–36), vol. III, pp. 213–222.

[11] This refers to the pressure that would normally be reached by a continuous adiabatic expansion to the given exit area of the nozzle, that is, the pressure computed from Eq. (31), when f is the exit area. The "area ratio" ϵ is discussed in SEC. 9.

Magnitude of the Reaction Thrust

9. Calculation of the Net Rocket Thrust

If a rocket were to operate a vacuum, the thrust would be simply calculated from the equation $F=\dot{m}v$, with corrections for the fact that the exhaust stream might be slightly divergent rather than parallel. However, since it is usually immersed in the atmosphere and subject to pressure forces, a more detailed analysis is necessary.

The net thrust force F, regarded as positive when acting in a direction *opposite* the gas velocity, is the vector sum of all pressure forces over the inside and outside surfaces of the solid shell, or

$$\mathbf{F}=\int p d\mathbf{S}=\int_{S_i} p_i d\mathbf{S}_i+\int_{S_0} p_0 d\mathbf{S}_0, \tag{41}$$

where p is the magnitude of the pressure on the motor wall and $d\mathbf{S}$ is a vector element of surface area of the wall. Subscripts i and 0 refer to the inside and outside surfaces, respectively. From symmetry it is clear that the vector integral will be directed along the axis of the motor.

The integral over the outside surface S_0 may be evaluated as follows. The resultant force due to the uniform atmospheric pressure p_0 on a completely closed vessel at rest is zero. If this force is resolved into (i) the force acting on the plane area f_e of the nozzle exit section and (ii) all other external pressure forces on the rocket, we have [Fig. 6(a)]

$$p_0 f_e+\int_{S_0} p_0 d\mathbf{S}_0=0. \tag{42}$$

The effect of the open area f_e is to create an unbalanced force opposed to the thrust of magnitude $-p_0 f_e$, which is thus the value of the integral in Eq. (42).

The term $\int_{S_i} p_i d\mathbf{S}_i$ in Eq. (41) may be evaluated by considering the mass of gas contained in the rocket motor [Fig. 6(b)]. The momentum theorem requires that the integral of all the forces acting over a surface enclosing this mass be equal to the rate of flow of momentum through this surface, or $\dot{m}v_{ex}$, where v_{ex} is the *average* velocity along the axis of symmetry at the exit section. The pressure acting on the gas is $-p_i$ (the reaction to the pressure p_i on the motor wall), and the average pressure opposing the flow through the exit section of area f_e is p_e (note that p_e, the pressure at the nozzle exit, is not necessarily equal to p_0). Equating the total force on the gas to its time-rate of change of momentum, one obtains

$$-\int_{S_i} p_i d\mathbf{S}_i+p_e f_e=-\dot{m}v_{ex}. \tag{43}$$

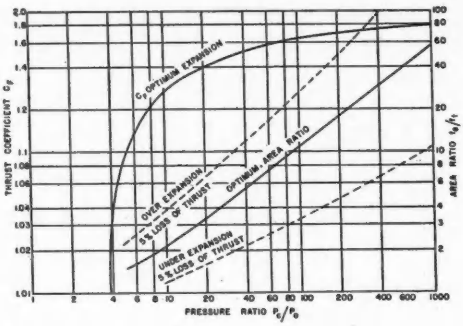

FIG. 7. Graph of theoretical maximum nozzle thrust coefficient C_F and area ratio ϵ as functions of pressure ratio for an ideal gas, with $\gamma=1.25$; C_F approaches 2.1 asymptotically as ϵ increases.

The negative sign is used in the right-hand member because it is customary to substitute the absolute positive numerical magnitude for $\dot{m}$, which is an inherently negative quantity, being the rate of *decrease* of total mass.

The average velocity v_{ex} in the axial direction is less than the true velocity of efflux v_e, which has a component perpendicular to the axial direction. A factor λ is applied to correct for this divergence of flow; thus we write $v_{ex}=\lambda v_e$. The value of λ depends on the angle of divergence of the nozzle, and can be calculated.[12] For example, if the half-angle of the nozzle is 15°, λ is 0.985.

The thrust equation (41) may now be evaluated by means of Eqs. (42) and (43); we obtain

$$F=\lambda\dot{m}v_e+(p_e-p_0)f_e. \tag{44}$$

The two terms in the right-hand member of Eq. (44) are sometimes referred to as *velocity thrust* and *pressure thrust*, respectively. If $p_e=p_0$, all the thrust is velocity thrust, and it can be shown, by evaluating v_e by means of Eq. (24) and differentiating F with respect to p_e, that F is also a maximum under these conditions. A nozzle for which $p_e=p_0$ is said to be "perfectly" expanded, and the value of the corresponding area ratio $\epsilon[=f_e/f_t]$ may be calculated; it is

$$\epsilon=f_e/f_t=\Gamma'\left\{\gamma\left(\frac{p_0}{p_c}\right)^{1/\gamma}\left(\frac{2}{\gamma-1}\right)^{\frac{1}{2}}\times\left[1-\left(\frac{p_0}{p_c}\right)^{(\gamma-1)/\gamma}\right]^{\frac{1}{2}}\right\}^{-1}. \tag{45}$$

Equation (45) is derived by substituting $\dot{m}$ from Eq. (39) in Eq. (31) for the area f and simplifying with the aid of Eq. (17) written in the form $p/\rho=R_sT$. Equation (45) is useful in practical design calculations.

If $p_e<p_0$ the gases are "overexpanded" and the pressure thrust will be negative, although partially compensated by an increased velocity thrust. If $p_e>p_0$ the gases are "underexpanded." Although the pressure thrust will then be positive—that is, in the same sense as the velocity thrust—it will not compensate completely for loss in v_e due to inadequate expansion. Owing to the fact that pressure and velocity thrusts tend to compensate, the net thrust F is rather insensitive to variations in the area ratio ϵ. For example, a nozzle correctly expanded at sea level gives about 6 percent less thrust at an elevation of 40,000 ft than one which is correctly designed for that altitude.

10. Performance Parameters Useful in Design

(*a*). *Effective exhaust velocity.*—It is difficult to determine v_e and p_e of Eq. (44) experimentally. Furthermore, because the expansion is not strictly adiabatic, frictionless and "perfect," an *effective* exhaust velocity c is used, defined by the equation

$$c\equiv\lambda v_e+(p_e-p_0)f_e/\dot{m}, \tag{46}$$

so that Eq. (44) may be written in the form

$$F=\dot{m}c. \tag{47}$$

The quantity c was discussed from another point of view in connection with Eq. (3). It is the parameter used in all practical design work, although it may differ from the true velocity v_e. It is determined in practice by measuring thrust F and mass flow rate $\dot{m}$ and using Eq. (47). The F measured in Eq. (47) includes pressure, friction and divergence effects, and may differ from the F due to reaction alone, as defined in Eq. (1).

(*b*). *Thrust coefficient.*—It is found useful in practice to define the thrust of a rocket in the alternative form

$$F=C_{FX}p_cf_t. \tag{48}$$

Since all the quantities involved in Eq. (48) are readily measurable, experimental values of the experimental thrust coefficient C_{FX} may be found that can then be applied to determine the throat area of a rocket having any desired thrust and

[12] See reference 2, p. 449.

chamber pressure. A theoretical expression for the thrust coefficient which may be compared with this experimental value is found as follows.

If the thrust [Eq. (44)] is expressed in terms of pressure by means of Eqs. (24) and (39), one obtains, assuming parallel flow [$\lambda = 1.0$],

$$F = \Gamma' \left\{ \frac{2}{\gamma - 1} \left[1 - \left(\frac{p_e}{p_c} \right)^{(\gamma-1)/\gamma} \right] \right\}^{\frac{1}{2}} p_c f_t + (p_e - p_0) f_e. \quad (49)$$

Dividing this equation by $p_c f_t$, one arrives at an expression for the theoretical thrust coefficient without divergence correction,

$$C_F = \frac{F}{p_c f_t} = \Gamma' \left\{ \frac{2}{\gamma - 1} \left[1 - \left(\frac{p_e}{p_c} \right)^{(\gamma-1)/\gamma} \right] \right\}^{\frac{1}{2}} + \left(\frac{p_e - p_0}{p_c} \right) \frac{f_e}{f_t}. \quad (50)$$

This coefficient C_F has a maximum value for the case of a "perfectly" expanded nozzle in which $p_e = p_0$.

A chart of the value of maximum C_F and corresponding optimum area ratio ϵ for the typical case of $\gamma = 1.25$ is given in Fig. 7. These are the values that obtain when $p_e = p_0$. In some applications, such as long-range missiles, the pressure ratio p_c/p_0 varies considerably. It is not possible for the area ratio of a fixed nozzle to be correct for more than one pressure ratio, since a "rubber" nozzle—one for which ϵ is variable—cannot be constructed without undesirable complications. A compromise value of ϵ is often chosen such that ϵ is correct at the altitude reached when half the propellant is consumed.

In a real nozzle with divergence and friction the measured C_{FX} of Eq. (48) is lower than C_F of Eq. (50). For a frictionless and correctly expanded nozzle we should have $C_{FX} = \lambda C_F$ [see Eq. (46)]; however, experimentally measured values of C_{FX} are 2 to 4 percent lower than the value of C_{FX} so calculated, owing to friction and other losses.

(*c*). *Characteristic velocity.*—In addition to the thrust coefficient C_{FX} it is useful to have a practical empirical parameter by which the mass flow rate $\dot{m}$ may be calculated, since Eq. (48) gives no information about $\dot{m}$. We may rewrite Eq. (39) for the mass flow rate in such a way as to define a new parameter c^*, called the *characteristic velocity;* thus,

TABLE I. Summary of performance parameters.

Parameter	Symbol	Definition	Unit	Typical value	Range of values
Specific propellant consumption	$\dot{w}_{sp}$	$\dot{w}/F$	sec^{-1}	0.0051	0.0100–0.0036
Effective exhaust velocity	c	$Fg/\dot{w}$	ft sec^{-1}	6300	3300–9000
Specific impulse	I_{sp}	$Ft/W = c/g$	sec	196	100–380
Nozzle coefficient	C_{FX}	$F/p_c f_t$	—	1.36	1.1–1.8
Characteristic velocity	c^*	$p_c f_t g/\dot{w}$	ft sec^{-1}	4630	3000–5000

$\dot{w}$, rate of propellant consumption (lb/sec);
F, thrust (lb);
g, 32.2 ft/sec^2;
W, total propellant weight (lb);
t, total duration of thrust (sec);
p_c, chamber pressure (lb/in.^{2});
f_t, nozzle throat area (in.^{2}).

$$\dot{m} = \frac{\Gamma'}{a_c} p_c f_t = \frac{p_c f_t}{c^*}, \quad (51)$$

where

$$c^* \equiv \frac{p_c f_t}{\dot{m}} = \frac{a_c}{\Gamma'} = \frac{(\gamma R T_c / M)^{\frac{1}{2}}}{\Gamma'}. \quad (52)$$

The quantity c^* as defined by Eq. (52) has the convenient property that we may express the effective exhaust velocity c in terms of it and the thrust coefficient C_{FX}; thus,

$$c = F/\dot{m} = \frac{F/p_c f_t}{\dot{m}/p_c f_t} = C_{FX} c^*. \quad (53)$$

Then, when c^* has been measured experimentally for a given propellant burning at a definite pressure, both mass flow rate $\dot{m}$ and effective exhaust velocity c may be calculated for that propellant. It may be seen from Eq. (52) that c^* is determined only by properties of the propellant and the throat diameter. Thus it is independent of exit conditions and may be considered as the parameter indicating the efficacy of the gas generation or combustion process. The quantity c^* is commonly used as a measure of the merit of the propellant, although its value is affected also by combustion-chamber design. By using physico-chemical methods to calculate T_c, γ and M in Eq. (52), a theoretical value of c^* may be found; it is about 10 percent higher than the experimental value calculated from measurements of $p_c f_t/\dot{m}$.

(*d*). *Summary of motor performance parameters.* —Besides the performance parameters c, C_F and c^*, there are two more parameters, first introduced in SEC. 2, which can now be expressed in terms of the newer quantities. The propellant consumption of a rocket motor is expressed by the *specific propellant consumption* $\dot{w}_{sp}$ in pounds of propellant consumed per second, per pound of thrust, or

$$\dot{w}_{sp}=\dot{m}g/F=g/c=g/c^*C_F. \tag{54}$$

The reciprocal of the specific propellant consumption is known as the *specific impulse*, or *performance index*, I_{sp}, ordinarily expressed in pounds thrust per pound of propellant consumed per second, or

$$I_{sp}=F/\dot{m}g=c/g=c^*C_F/g. \tag{55}$$

A typical value and range of values for each of these parameters are listed in Table I. An examination of this table will give a certain perspective of rocket performance.

II. SOLID-PROPELLANT ROCKETS

Characteristics of Solid-Propellant Rockets

11. Applications of Solid-Propellant Rockets

To those who were not directly concerned with rockets during the war, the term "rocket" is likely to bring to mind what are more technically described as artillery rockets, that is, rocket-propelled missiles. Most artillery rockets are propelled by rocket motors using solid propellants. The term "solid propellant" arises from the fact that the propellant, before its combustion, has mechanical properties similar to those of a solid body. The much publicized Bazooka is an artillery rocket of this type. The German V-2 is an artillery rocket, but it is a liquid-propellant rocket.

Rockets, as war weapons, are not new. Sir William Congreve is credited with developing, early in the nineteenth century, a variety of successful war rockets. These rockets were used with devastating effect by British soldiers in the siege of Copenhagen in 1807. However, during the second half of the nineteenth century the art of gunnery was so successfully developed that cannon soon became far superior to rockets in both range and accuracy. The question then arises, why was there a great effort made in artillery-rocket development in World War II? The answer is that the emphasis placed upon mobility, firepower and the use of aircraft in the recent war has again made the artillery rocket an important weapon.

Another development which recently received serious attention was the application of rockets to aircraft, either as boosters or as the sole means of propulsion. Solid-propellant rockets, which can efficiently supply a given thrust for only a limited time, are not especially well suited for the latter purpose. However, solid-propellant rockets of a special type were developed that could fill the important need of assisting the take-off of the faster and heavier aircraft, which were requiring longer and longer runways. This assisted take-off rocket (called JATO for jet assisted take-off) was especially important in aircraft-carrier operation, where the length of runways is necessarily quite restricted.

The artillery rocket is a much more mobile weapon than the gun. The rocket launcher is merely an aiming device; it does not have to confine the propelling gas as does a gun tube, nor does it have to absorb a large recoil. Consequently the launcher for an artillery rocket has about the same weight as one round of ammunition. This is in strong contrast to the 75-mm gun, for example, which weighs 2600 lb and fires a 14-lb missile. Even though artillery rockets are still inferior to guns, both in range and in accuracy, the more mobile rocket can be used from positions farther forward and does not have to fulfill such rigid requirements.

For the first time the infantry can carry with it a weapon—the Bazooka—capable of stopping a tank at moderate range. A medium aircraft can fire a barrage of 5-in. rockets equivalent to a destroyer's broadside, and after launching its rockets fly just as well as a sister ship that was never equipped for rockets at all. The problem of absorbing the recoil from a 5-in. gun without damage to the frail aircraft structure is a difficult one, and the reduction in aircraft performance entailed in carrying such a weapon would be considerable. Thus we see why the artillery rocket is so important as a supplementary weapon to the gun in modern mobile warfare.

The fundamental characteristic of solid-pro-

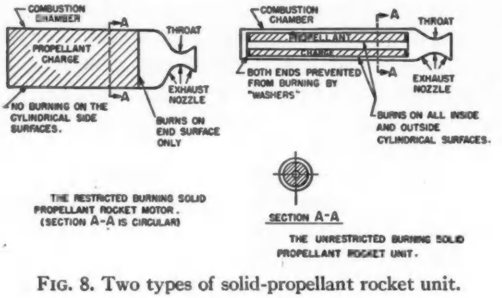

FIG. 8. Two types of solid-propellant rocket unit.

pellant rockets as compared to liquid-propellant rockets is greater simplicity—in manufacture, in operation, and in servicing in the field. The result of this simplicity is that if the required duration of a run is less than about 30 sec, a solid-propellant rocket can be made that has a lower weight than a liquid-propellant rocket which delivers the same thrust. This simplicity makes the manufacture and use of artillery rockets feasible, and gives solid-propellant rockets a definite advantage in application to assisted take-off and to several other rocket-propulsion problems. Furthermore, the advantage of simplicity is important in many cases when a handy booster of some type is required as a purely auxiliary device.

This advantage of simplicity is somewhat offset by a sensitivity of performance of solid-propellant rockets to climatic conditions, particularly ambient temperature—an effect that is usually unimportant for liquid-propellant rockets. Another disadvantage is lack of controllability. For artillery rockets the duration is so short—1 sec or less—that control of the thrust during operation is unimportant. For assisted-take-off rockets, with durations up to 30 sec, control of the rocket to the extent of turning it on, off, and on again would be valuable; however, such control is impractical, although a device sufficiently simple to be practical can be made that allows shutting off the rocket after it is ignited. Control of the thrust from a given solid-propellant rocket during its operation, in a practical manner, is not possible.

Thus we see that the most valuable and important characteristic of the solid-propellant rocket is simplicity. In situations that require controlled operation a liquid-propellant rocket is better. Furthermore, any "improvement" of a solid-propellant rocket that makes it a complicated mechanical device is usually not desirable. As we shall see, improvements in solid-propellant rockets should take the fundamental direction of better propellants and better materials of construction, and, for artillery rockets, improved accuracy.

12. Manner of Operation

A solid-propellant rocket unit consists of a charge of solid propellant within a combustion chamber, and an exhaust nozzle through which the products of combustion escape. The reaction due to the expulsion of the combustion products is the thrust of the rocket. The propellants used in rockets, just as those used in guns, do not explode but instead burn away at a definite rate on those surfaces that are exposed to the hot gas or flame within the combustion chamber. The rate r at which the surface of the propellant recedes in a direction normal to itself during burning is called the *burning rate* and is usually expressed in inches per second. The burning rate depends upon chamber pressure, increasing with higher pressures; but for most solid propellants in use today, the burning rates for a pressure of 2000 lb/in.^{2} lie between 1 and 2 in./sec.

Now the thrust of a rocket motor [Eq. (1)] is equal to the product of exhaust velocity v and mass flow rate $\dot{m}$, so that in order to get a large thrust a large burning surface must be used to obtain a large mass flow. Similarly, the duration of the thrust is determined by the burning rate. Since a given combustion chamber can contain only a limited amount of propellant, the thrust may be made large for a short time by providing a large burning surface, and conversely. A wide variety of arrangements of solid-propellant charges have been used in solid-propellant rocket units. Here we have space to discuss only two extreme types, the restricted-burning and the unrestricted-burning rocket, and we will give a somewhat detailed picture only for the simpler restricted-burning unit. In SEC. 20 we will describe in a qualitative way the special problems that arise in the case of the unrestricted-burning solid-propellant rocket unit. Diagrammatic sketches of the two extreme types of rocket are shown in Fig. 8.

In the restricted-burning rocket, the propellant charge is made in the form of a solid right circular cylinder. The cylindrical side surfaces and one end face are prevented from burning by a suitable lining or coating, and burning is allowed to proceed from one end only. This type of rocket is sometimes called "end burning," or "cigarette burning." The duration of thrust obtained from a restricted-burning rocket is proportional to the length of the charge and also depends upon the chamber pressure and the type of propellant used. The thrust obtained from such a rocket is proportional to the area of the circular burning surface, and also depends upon the chamber pressure, the type of propellant used, and the efficacy of the rocket-unit design.

In the unrestricted-burning rocket the charge is often in the form of a hollow right circular cylinder (tubular charge, or tubular "grain"). This charge is held in place by a suitable grid, or trap at the rear, or exhaust-nozzle, end and by a few centering support points distributed along its length, but is otherwise uninhibited. (The annular end surfaces are sometimes presented from burning by inhibiting "washers.") The charge is ignited and allowed to burn on all surfaces except for very small areas at support points. The thrust from such a unit is proportional to the total burning area and depends upon the chamber pressure, the type of propellant used, and the design of the rocket unit and propellant grain. The duration is proportional to the thickness of the cylindrical wall (web thickness) and depends upon the chamber pressure, the type of propellant, the internal geometry of the combustion chamber and the geometry of the propellant grain.

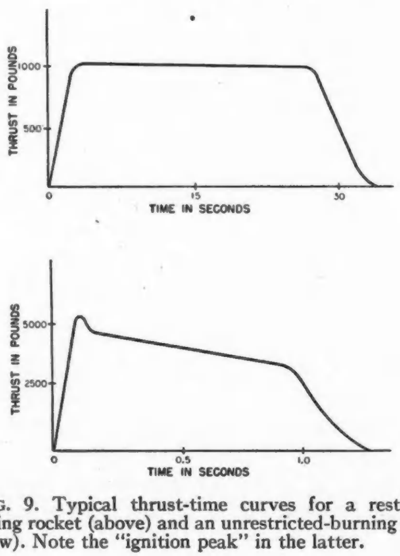

FIG. 9. Typical thrust-time curves for a restricted-burning rocket (above) and an unrestricted-burning rocket (below). Note the "ignition peak" in the latter.

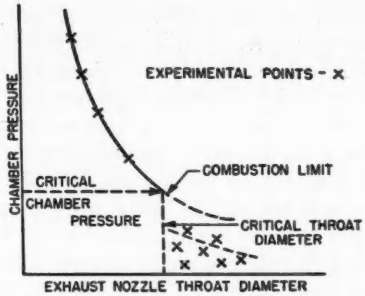

FIG. 10. Curve to show the meaning of the combustion limit.

The chamber pressure generated by either type of rocket unit is a function of the ratio of the burning area to the cross-sectional area of the exhaust-nozzle throat. This ratio is sometimes called the "area ratio"* K of the rocket. For unrestricted-burning rockets the chamber pressure varies considerably from one end of the grain to the other and depends in a marked degree upon the geometry of the propellant charge and the internal geometry of the combustion chamber.

Typical thrust-time performance curves for the two types of rocket are shown in Fig. 9. The thin web and large burning surface of the unrestricted-burning rocket favor designs requiring large thrust and short duration, while for similar reasons the converse is true for the restricted-burning rocket.

13. Special Characteristics

There are a number of special characteristics of solid propellants, a knowledge of which is important in understanding their behavior. These characteristics constitute some of the major limitations of the solid rocket propellants now in use. They are: (i) temperature sensitivity; (ii) temperature limits; (iii) the combustion limit; (iv) the pressure limit; (v) decomposition on storage.

Temperature sensitivity.—If a number of iden-

tical rocket units are conditioned by storage at different temperatures, it is found when they are fired that those stored at high temperatures operate at higher chamber pressure and thrust than those stored at low temperatures. The duration for the high temperature group is shorter than that for the low temperature group, but the total impulse for any unit is almost the same. We conclude that temperature has an important effect on the rate processes within the unit, but only a minor effect on the total energy or impulse. The *temperature sensitivity* α is defined quantitatively by the equation

$$\alpha=\frac{1}{p_c}\left(\frac{dp_c}{dT_p}\right)_K, \tag{56}$$

where p_c is the chamber pressure and T_p is the temperature of the propellant charge before firing. For restricted-burning ballistite the temperature sensitivity is 0.0058 (deg F)$^{-1}$; for unrestricted-burning ballistite it is much more—0.0132 (deg F)$^{-1}$. For composite asphalt-potassium perchlorate propellants developed in our laboratory,[13] and for some special composite propellants developed by the National Defense Research Committee (NDRC) during the war the temperature sensitivity is only 0.0024 (deg F)$^{-1}$. The temperature sensitivity of ballistite is so large that when it is utilized in unrestricted-burning rockets dangerously high pressures above 120°F and poor combustion below 0°F limit the use of such rockets to about the temperature range indicated. The temperature sensitivity of the GALCIT and NDRC propellants is small enough so that it is not usually considered objectionable. In the case of unrestricted-burning artillery rockets large temperature sensitivity impairs accuracy.

Temperature limits.—Special limitations on the temperature range within which a rocket may be used are sometimes introduced by a change of mechanical properties of the propellant with temperature. Certain types of the GALCIT propellants may soften and deform sufficiently when stored at high temperature so that an abnormally large burning area is exposed. Upon ignition the resulting high chamber pressure may cause failure of the unit. Ballistite may soften sufficiently at high temperatures so that the large pressure gradient along the grain which is present in unrestricted-burning units may cause the charge to break up, exposing a large burning area and leading to failure of the unit.

At low temperatures certain types of the GALCIT propellants may become so embrittled that the charge will shatter upon ignition of the unit. The large burning area exposed in a failure of this type may lead to a violent explosion. The NDRC composite propellants do not seem to be subject to temperature limitations due to change of the mechanical properties of the propellant with temperature.

Combustion limit.—If a number of rockets, identical except for the exhaust-nozzle throat diameter, are fired, a graph can be plotted showing chamber pressure as a function of throat diameter (Fig. 10). It is found, as might be expected, that the pressure decreases as the throat diameter increases. However, when a certain throat diameter is exceeded, the chamber pressure is found to be far below the pressure predicted from an extrapolation of the high pressure portion of the curve. The pressure corresponding to this critical thrust diameter is called the *combustion limit.* If a number of units are fired with this same large throat diameter the chamber pressure will vary erratically from unit to unit. Finally, if the exhaust-nozzle throat is made large enough individual units will burn not continuously but in an irregular manner with a chugging noise. This last phenomenon is called "chuffing."

Thus it appears that a given propellant cannot be used at an arbitrarily low pressure, and the designer of a rocket unit must assume chamber pressures above the combustion limit if reproducible performance from unit to unit is to be obtained. When low over-all rocket weight is desirable, a high combustion limit is a serious disadvantage, since the weight of the walls of the combustion chamber is directly proportional to its volume and to the design chamber pressure.

The combustion limit of ballistite is about 500 lb/in.^{2}; that of GALCIT propellants is about 1000 lb/in.^{2}. The NDRC composite propellants have a combustion limit not exceeding 100 lb/in.^{2}.

[13] These will usually be designated as GALCIT propellants, GALCIT being an abbreviation for *G*uggenheim *A*eronautical *L*aboratory, *C*alifornia *I*nstitute of *T*echnology.

The combustion limit of the straight nitrocellulose propellants frequently used in guns exceeds 5000 lb/in.[2]; therefore such propellants are not suitable for rockets. Ballistite as used for rockets, consisting of roughly equal parts of nitrocellulose and nitroglycerine, is essentially a modification of the gun propellants of the same name.

Pressure limit.—Some propellants may safely be used only below a critical chamber pressure. If the critical chamber pressure is exceeded, the propellant burns in a violent and unpredictable manner. Brittle propellants with a granular structure are particularly subject to this effect. Most of the commonly used rocket propellants have been developed to such a degree that the pressure limit exceeds 5000 lb/in.[2], and therefore is not a problem in rocket unit design.

Decomposition on storage.—The double-base[14] propellants (ballistite and related materials) slowly decompose with prolonged storage. Their decomposition is autocatalytic, and diphenylamine is usually added to neutralize the effect of the initial decomposition products. It is inadvisable to store ballistite at elevated temperatures for long periods of time.

The particular group of composite propellants developed by the NDRC that consist of ammonium picrate and sodium nitrate may become soft and mechanically weak owing to absorption of moisture from the atmosphere by the sodium nitrate. These propellants must be shipped in moisture-tight containers and must not be exposed to moisture before use. The GALCIT propellants seem to store indefinitely with no sign of chemical decomposition.

Theory of Solid Propellant Operation

14. The Burning-Rate Law

The ideal solid propellant rocket should have a thrust-time curve with a flat top. Upon ignition, the chamber pressure should rise rapidly but smoothly to a steady value, remain constant as the propellant charge burns away, and then fall off as the residual high pressure gas flows from the combustion chamber. For a given rocket, the

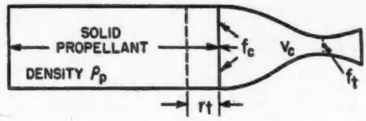

FIG. 11. Restricted-burning solid-propellant rocket, showing the notation used in the discussion of stability.

thrust and chamber pressure are nearly proportional over a wide range of pressures and one may speak of a "flat-top" performance curve with reference to either thrust or chamber pressure.

Irregular pressure-time curves are not desirable, even aside from difficulties which might arise in the application of units delivering varying thrust, because the strength and hence the weight of the combustion chamber must be designed to withstand the peak pressure. The exhaust velocity of a solid-propellant rocket is relatively insensitive to chamber pressure, whereas the weight of the unit is directly proportional to the pressure that it must withstand. The over-all weight of most present-day rockets could be reduced by a considerable factor if solid propellants with a lower combustion limit were available.

We will now discuss, within very narrow limits, the theory of the combustion process within a rocket unit. The discussion will be limited entirely to the restricted-burning rocket because it is in principle simpler; later (SEC. **20**) there will be indicated in a qualitative way the special problems of the unrestricted-burning rocket. In our discussion we will follow the work of von Kármán and Malina.[15]

As we have indicated, very little can be done in a practical way to regulate the operation of a solid-propellant rocket once it has been ignited. The parameters controlling the operation of the unit must be incorporated in its design, from the start. The burning-rate law describes in a purely empirical way the factors that affect the rate at which the solid propellant burns away in layers parallel to the burning surface. Some efforts have been made to obtain theoretical expressions for the burning-rate law based upon the heat of combustion, the specific heat, and other more

[14] The term "double-base" refers to propellants containing the two basic ingredients nitrocellulose and nitroglycerine, in contrast with single-base propellants which are chiefly composed of nitrocellulose.

[15] Th. von Kármán and F. J. Malina, unpublished report prepared in 1940.

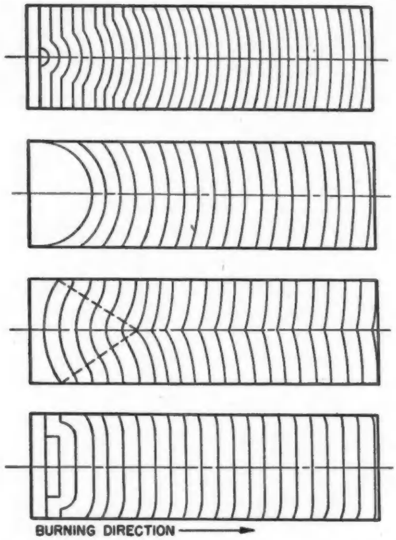

FIG. 12. Sketches illustrating burning-surface stability.

fundamental characteristics of the propellant. The combustion of a solid propellant is a very complicated process involving reactions in the solid phase, reactions in the liquid phase, if any, and reactions in the gas phase. For actual solid propellants the reactions in any one phase are also very complicated and take place under high pressure; moreover, the reactions in all the various phases that may be present at a given time are mutually interdependent. It has taken years of empirical study to find out the proper theoretical approach to the kinetics of the simple, low pressure, gas-phase, hydrogen-oxygen combustion process. Consequently, as one might expect, the theories of the solid-propellant burning rate are quite provisional and involve many approximations. Although these theories frequently give valuable insight into the combustion process, they are not sufficiently quantitative to furnish a satisfactory basis for investigation of the internal ballistics of the rocket.

From special experimental studies it has been found that the burning rate r of a solid propellant may be expressed as a function of the chamber pressure p_c, the temperature T_p of the propellant charge before ignition, the velocity v of gas flow parallel to the burning surface (this factor is not obviously present in restricted-burning rockets), and the time t measured from the instant of ignition; that is,

$$r = r(p_c, T_p, v, t). \tag{57}$$

For the restricted-burning rocket the dependence of r on v and t is small and we will use the expression

$$r = a p_c^n, \tag{58}$$

where a and n are experimentally determined parameters that depend upon the propellant under consideration. The parameter a is assumed to depend upon the temperature, and n is assumed to be independent of temperature. The value of n is between 0.4 and 0.8 for the solid propellants commonly used in rockets, and a is of such an order of magnitude that r is about 1 in./sec. when p_c is 2000 lb/in.^{2}.

15. Stability of the Shape of the Burning Surface

Consider the schematic restricted-burning rocket outlined in Fig. 11. Suppose that the burning surface, of area f_c, is not flat as indicated but, because of faulty preparation or because of combustion of an inhomogeneous region of propellant charge, it has become concave or of some other irregular shape. Will the burning surface flatten out as burning proceeds or will it become more and more distorted, leading to an excessively large burning area and ultimate failure of the unit? This question is important because, as we will see, a 10-percent increase in burning area can cause a 70-percent increase in chamber pressure.

If we assume that the burning surface moves normally to itself, and at the same rate at every point, then the sketches in Fig. 12 show that it tends to flatten out as burning proceeds. The assumption that the surface moves perpendicularly to itself at each point can be made plausible from considerations of symmetry. For restricted-burning rockets the velocity of gas flow is almost zero at all points of the surface and the pressure is nearly uniform, so that from Eq. (58) we should expect the burning rate to be the same at each point. Unrestricted burning will be considered in SEC. 20. The general conclusion is that the shape of the burning surface is stable, that is, the surface tends to remain flat as the burning proceeds.

16. The Fundamental Differential Equation

It will be shown presently that an equilibrium chamber pressure exists for solid-propellant rockets in the sense that, if the burning area of the charge remains constant as the charge burns away, then the chamber pressure remains constant during the burning period. Assuming that the burning area is constant one can write the following equation for the conservation of mass:

$$\left\{\begin{matrix}\text{Mass burned}\\ \text{per unit time}\end{matrix}\right\}=\left\{\begin{matrix}\text{Increase of}\\ \text{mass of gas}\\ \text{in combus-}\\ \text{tion cham-}\\ \text{ber per}\\ \text{unit time}\end{matrix}\right\}+\left\{\begin{matrix}\text{Mass flow-}\\ \text{ing out}\\ \text{through}\\ \text{exhaust}\\ \text{nozzle per}\\ \text{unit time}\end{matrix}\right\},$$

or, with the aid of Eq. (39),

$$rf_c\rho_p=\frac{d}{dt}(\rho_c V_c)+\Gamma(R_sT_c)^{\frac{1}{2}}f_t\rho_c. \qquad (59)$$

Here

$$\Gamma=\gamma^{\frac{1}{2}}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/2(\gamma-1)}=\frac{\Gamma'}{\gamma^{\frac{1}{2}}}; \qquad (60)$$

f_c is the area of the burning surface, assumed to be constant; ρ_c, the density of the gas in the combustion chamber; ρ_p, the density of solid propellant; V_c, the volume of gas in the combustion chamber, which will increase in time as the charge burns away; R_s, the engineering gas constant, equal to the universal gas constant divided by the average molecular weight of the propellant gas; f_t, the area of the exhaust-nozzle throat; γ, the ratio of specific heats at constant pressure and volume for the products of combustion; T_c, the temperature of the gas within the combustion chamber, sometimes called the "flame temperature"; and t, the time measured from the instant of ignition. In deriving Eqs. (59) and (60) it is assumed that the ideal gas law is valid for the products of combustion, and that p_c is in excess of the critical exhaust-nozzle discharge pressure.

Now if the combustion within the rocket takes place at constant pressure, then the temperature T_c will have a constant value given by the equation

$$c_pT_c=H_p, \qquad (61)$$

where c_p is the mean specific heat at constant pressure for the propellant gas, and H_p is the heat of combustion of the solid propellant at constant pressure. We neglect the effect on T_c of the initial propellant temperature. For ballistite, T_c is about 5000°F (5460°R); and for other propellants it is between 3000° and 4000°F. A rather complicated analysis shows that 10 msec after ignition T_c is within 5 percent of the value given by Eq. (61), and that it remains within a few percent of this value under large pressure fluctuations. Consequently we will assume that T_c is constant with respect to time. With this assumption we have

$$\frac{d}{dt}(R_sT_c\rho_cV_c)=R_sT_c\frac{d}{dt}(\rho_cV_c). \qquad (62)$$

We multiply Eq. (59) by R_sT_c, note the equation of state,

$$p_c=R_s\rho_cT_c, \qquad (63)$$

define p_p, a constant for a given propellant, by the equation

$$p_p=\rho_pR_sT_c, \qquad (64)$$

and finally obtain

$$rf_cp_p=\frac{d}{dt}(p_cV_c)+\Gamma(R_sT_c)^{\frac{1}{2}}f_tp_c, \qquad (65)$$

which is the same as Eq. (59) except that the densities have been replaced by pressures.

The constant p_p defined by Eq. (64) has the dimensions of pressure, and because ρ_p, the density of the solid propellant, is much larger than the density of the gas in the combustion chamber, p_p has numerical values of 100,000 to 200,000 lb/in.[2]. High values of p_p indicate large specific impulse for the propellent, and to a

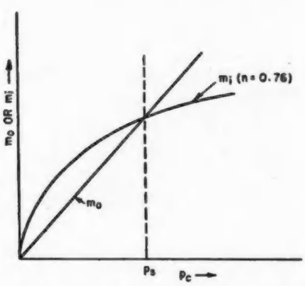

FIG. 13. Chamber-pressure stability curves, for $n=0.76$.

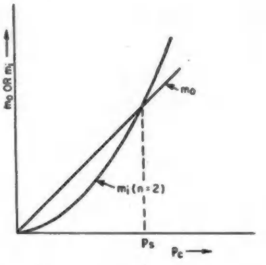

FIG. 14. Chamber-pressure stability curves, for $n=2$.

limited extent p_p represents an index of merit for solid propellants.

By the geometry indicated in Fig. 11, and the assumed constancy of f_c, we have

$$\frac{d}{dt}(p_c V_c) = V_c\frac{dp_c}{dt} + p_c\frac{dV_c}{dt} = V_c\frac{dp_c}{dt} + p_c r f_c. \qquad (66)$$

We can now write Eq. (65) in the form

$$\frac{dp_c}{dt} = \frac{1}{V_c}\left[r(p_p - p_c) - \Gamma(R_s T_c)^{\frac{1}{2}}\left(\frac{f_t}{f_c}\right)p_c\right]. \qquad (67)$$

Moreover V_c is given by the equation

$$V_c = V_c^0 + f_c\int_0^t r\,dt, \qquad (68)$$

where V_c^0 is the "free" volume within the combustion chamber which is not filled initially with solid propellant.

Equations (67) and (68) can be solved simultaneously by numerical methods, when r is known as a function of p_c, to give the variation of chamber pressure with time.

17. Stability of the Chamber Pressure

We note that when the coefficient of $1/V_c$ in the right-hand member of Eq. (67) is zero the chamber pressure does not vary with time. Let us call the corresponding value of the chamber pressure p_s. Now the algebraic sign of dp_c/dt is the same as that of the coefficient of $1/V_c$ since V_c is always positive.

Let the two terms in the coefficient of $1/V_c$ be denoted as follows:

$$m_i(p_c) = r(p_p - p_c) = (p_p - p_c)ap_c^n, \qquad (69)$$

$$m_0(p_c) = \Gamma(R_s T_c)^{\frac{1}{2}}\left(\frac{f_t}{f_c}\right)p_c. \qquad (70)$$

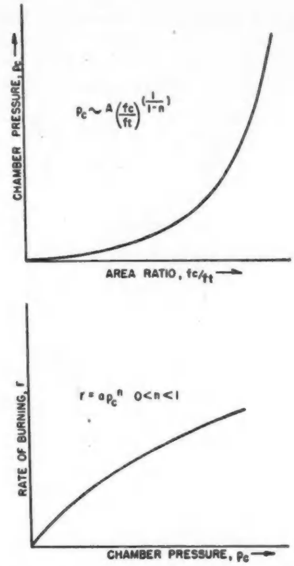

FIG. 15. Equilibrium chamber pressure-area ratio and burning rate-chamber pressure curves.

Most rocket motors will fail mechanically if their internal pressure exceeds 10,000 lb/in.^{2}; recalling the magnitude of p_p we see that we may write in place of Eq. (69),

$$m_i(p_c) = p_p a p_c^n. \qquad (71)$$

Now Eq. (67) shows that when m_i is larger than m_0, p_c is increasing; and when m_i is less than m_0, p_c is decreasing. In Fig. 13 m_i and m_0 are plotted as functions of p_c for a propellant for which $n=0.76$, a common value, and in Fig. 14 is plotted an example with $n=2$.

In Fig. 13 we see that if the chamber pressure exceeds p_s by a small amount, then m_0 exceeds m_i and dp_c/dt becomes negative, so that the pressure returns to p_s. Similarly, if p_c should fall below p_s then m_i is greater than m_0 and the pressure rises again to p_s. Thus the chamber pressure is stable for small disturbances.

In Fig. 14, on the other hand, we see that if p_c exceeds p_s, then m_i exceeds m_0 and dp_c/dt is positive, so that the pressure rises still further. Conversely, if p_c is less than p_s the pressure eventually falls to zero. Thus if $n=2$, p_s is a point of unstable equilibrium.

These special cases hold for all values of the area ratio f_c/f_t in Eq. (70), since as this ratio is

varied one gets a family of straight lines through the origin. From elementary calculus we know that when $n>1$ the curve for m_i is convex towards the p_c-axis, and when $n<1$ the curve is concave towards this axis. Consequently our argument holds in general, and we say that the chamber pressure is stable if n, the exponent in the burning-rate law, is less than 1.0, and unstable if n is greater than 1.0. In theory, owing to the term p_p-p_c in Eq. (69), which we have here set equal to p_p, the limiting case for which $n=1$ is also stable. However, for practical rockets, propellants with a burning-rate exponent n exceeding 0.85 are so sensitive to minor variations in preparation as to be unreliable and dangerous.

18. The Equilibrium Chamber Pressure

If we now assume that n is less than 0.85, then we know that a stable chamber pressure p_s (which we will call p_c from now on to conform to standard notation) exists.[16] This value is obtained by setting $dp_c/dt=0$ in Eq. (67). This gives for the equilibrium chamber pressure the equation

$$f_c/f_t=\frac{\Gamma(R_sT_c)^{\frac{1}{2}}p_c}{r(p_p-p_c)}=\frac{\Gamma(R_sT_c)^{\frac{1}{2}}}{a(p_p-p_c)}p_c^{1-n}. \qquad (72)$$

A typical curve showing the variation of chamber pressure with area ratio f_c/f_t, sometimes called K, is shown in Fig. 15, along with a burning-rate curve. As would be expected, higher pressures are obtained with larger area ratios, that is, with smaller exhaust-nozzle openings.

For the older propellants the combustion limit is such that the design point for a rocket unit was on a rather steep part of the area-ratio curve. An approximate solution of Eq. (72) for p_c in terms of f_c/f_t shows that p_c varies as $(f_c/f_t)^{1/(1-n)}$, and when $n=0.8$ there results a very steep fifth-power law. This illustrates the delicate balance between gas evolution within the rocket motor from the combustion of propellant, and the escape of gas through the exhaust nozzle. One should realize that this is a dynamic balance depending upon a rate process, the burning-rate law, and that minor variations in the properties of the propellant can have major effects on the burning rate and yet cause no detectable change in the heat of combustion, specific gravity or other properties often used to control the preparation of gun propellants. Consequently, special tests are needed to control the preparation of rocket propellants.

Assuming that the area ratio f_c/f_t is constant, and neglecting p_c relative to p_p, one may differentiate Eq. (72) with respect to the temperature of the propellant T_p (recall that a is assumed to be a function of T_p); one obtains

$$\frac{1}{p_c}\left(\frac{dp_c}{dT_p}\right)_K=\frac{1}{1-n}\cdot\frac{1}{a}\cdot\frac{da}{dT_p}. \qquad (73)$$

Here again the factor $1/(1-n)$ appears as a magnification factor. For one might think of $(1/a)(da/dT_p)$ as a measure of the intrinsic temperature sensitivity of the propellant, but in a rocket, if $n=0.8$, say, this effect is increased fivefold. Some clever chemical modifications of

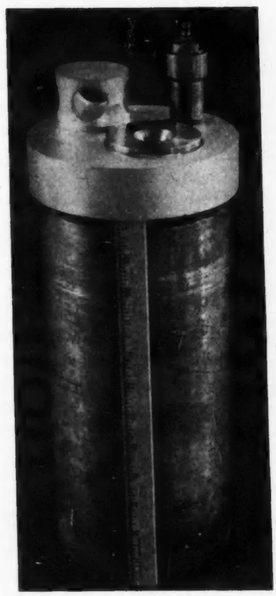

FIG. 16. A typical solid-propellant rocket (JATO), having a 200-lb thrust and 8-sec duration, for assisted take-off of aircraft, showing the nozzle, ignitor and safety rupture disk.

[16] We have said nothing about how quickly the chamber pressure returns to the equilibrium value if displaced from it. A simple approximate solution of Eq. (67), assuming V_c to be constant, gives $\Delta p_c(t)=\Delta p_c$ (initial instant) $\exp[-(p_p/p_s)kt/(V_c/f_c)]$, where k is a constant of value 0.2 in./sec. The recovery time is about 0.2 sec.

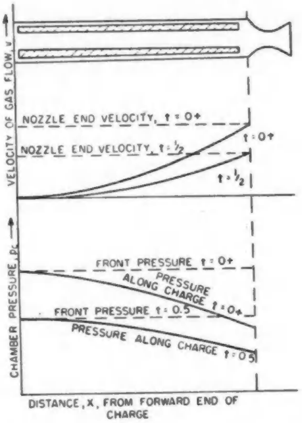

FIG. 17. General nature of velocity and pressure distributions within an unrestricted-burning solid-propellant rocket, shortly after starting, and when burning is half completed.

solid propellants are known that will reduce n from 0.76 to 0.45.

We conclude that a solid propellant will be satisfactory as a rocket propellant if the exponent in the burning-rate law lies between 0 and 0.85, and that the shape of the burning surface is stable.

19. Design of a Restricted-Burning Rocket

A brief outline of a satisfactory design procedure may help to give an intuitive "feel" for the character of a solid-propellant rocket. We assume that the designer is given the required thrust F and duration t_b, sometimes called the "burning time." He is then supposed to have selected a suitable propellant for which he has an experimental burning-rate curve, and also an experimental[17] area ratio-chamber pressure curve which will be similar to the curves in Fig. 15. Knowing the combustion limit for the propellant, the designer selects a pressure slightly above the combustion limit. Low pressures are desirable, since they permit reduction in weight and increase in safety. From experimental data he knows the specific impulse $I_{sp}(p_c)$ of the propellant for this design pressure p_c.

The weight of the propellant that must be used is then

$$W_p = Ft_b/I_{sp}. \tag{74}$$

From the experimental burning-rate curve (Fig. 15) the designer also knows $r(p_c)$, so that he knows the length of the propellant charge l_p from the equation

$$l_p = rt_b. \tag{75}$$

[17] Although our simple theory predicts correctly the general shape of the area-ratio curve, it is based upon the assumption of the ideal gas law, especially for gas flow from the exhaust nozzle. Hence an experimental curve is needed.

Knowing the density of the solid propellant ρ_p, he calculates the end area f_c and the diameter of the charge d_c, from the equations

$$l_p f_c \rho_p = W_p \tag{76}$$

and

$$\pi/4 d_c^2 = f_c. \tag{77}$$

From the design pressure p_c, the designer knows the area ratio K required to give this pressure (Fig. 15); then

$$f_t = f_c(1/K), \tag{78}$$

and

$$\pi/4 d_t^2 = f_t. \tag{79}$$

The remainder of the problem has to do with design of the exhaust nozzel, which has been outlined in SECS. 8 and 9, and the design of metal parts. Special design problems also arise because of the high temperature of the products of combustion, but these cannot be discussed here. A typical JATO unit (SEC. 11) is illustrated in Fig. 16.

20. Special Problems of Unrestricted-Burning Rockets

The propellant within an unrestricted-burning rocket burns over its whole surface; therefore, the products of combustion must escape by flowing past the burning charge. Clearly the chamber pressure must vary along the length of the charge, being highest at the forward end, and lowest at the rear, or exhaust-nozzle, end of the charge. The mean velocity of gas flow parallel to the charge will be zero at the forward end and will increase toward the exhaust nozzle. This is outlined in Fig. 17. In addition, as the charge burns away, the general pressure level within the rocket decreases since more space is available for passage of gas along the grain. The problem is no longer a steady-state problem.

For well-designed rockets the burning rate is almost the same at every point on the charge surface because, although higher pressure makes the charge burn faster at the forward end, the higher velocity of gas flow at the rear accelerates the burning rate to almost balance the effect of pressure. Clearly to design a rocket well in this sense is difficult. Good design is necessary since an unevenly burned charge will break up and

TABLE II. General characteristics of solid propellants.

Propellant	Flame temperature (°F)	Exhaust velocity (ft/sec)	Specific impulse (sec)	Burning rate at 1500 lb/in.² (in./sec)	Temperature sensitivity	Density (lb/ft³)
Ballistite	5000	7000	200	0.7	high	100
NDRC*	3000–4000	5500	180	0.2–1.0	low	115
GALCIT*	3000–4000	5500	180	1.4	low	115

* Copious quantities of smoke in the exhaust jet, a disadvantage in some applications.

be ejected as unburned slivers before its full impulse has been delivered to the rocket. Even good rockets may lose 5 percent of their charge in this manner.

A long range artillery rocket should be designed with a small diameter to reduce drag. But to put a given amount of propellant in a long narrow tube means that the charge will tend to "choke off" the gas flowing along the grain, owing to narrow wall and bore clearance. If this "choking" is so great as to make the gas flow reach its sonic velocity at some point of the burning charge the rocket is almost certain to function badly, if it does not explode. Thus the requirements of internal and external ballistics are directly opposed to each other.

Finally, since the weight of a pressure vessel is proportional to its volume and its design pressure, it is desirable to put as much charge as possible into a given combustion chamber. The choking effect encountered from adding too much charge limits the volumetric efficiency (ratio of propellant volume to total volume) of the rocket to something less than 75 percent.

The effects of temperature sensitivity are greatly aggravated by the geometry of the unrestricted-burning rocket, and although clever geometric design of the charge can reduce this effect, the restricted-burning rocket seems to be less temperature sensitive than an unrestricted-burning rocket utilizing the same propellant.

In general, the important problem for solid-propellant rockets of all sorts is a deeper understanding of the burning mechanism of solid propellants so that the burning rate may be controlled to improve the geometry of rocket design and reduce temperature sensitivity.

21. Solid Propellant Chemicals

Solid propellants for rockets are crudely classified as (i) homogeneous and (ii) composite. The most important homogeneous materials are ballistites, consisting of roughly equal parts of nitrocellulose and nitroglycerine. Composite materials include two types of propellant developed by the NDRC: one consists of ballistite modified by the addition of large amounts of inorganic salts to reduce the temperature sensitivity of the propellant; the other, of various modifications of a mixture of equal parts of pulverulent ammonium picrate and sodium nitrate molded together under high pressure with about 10 percent of an artificial resin binder. The early GALCIT propellants consisted of 75 percent pulverulent potassium perchlorate (oxidizer) mixed with 25 percent asphalt (fuel); this material was mixed and poured into the combustion chamber of a rocket while hot, and then cooled to a tough mass something like paving tar. General characteristics of the propellants are listed in Table II.

(*This is the first of three articles on the physics of rockets.*)

Biophysics

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The borderland in which physics and the biological sciences meet is rather broadly and loosely called *biophysics*. The increasing interest of physicists in this field has been evident for some time, and the trend in that direction appears to have been accelerating since the close of the war. It seems appropriate, therefore, to consider some general aspects of biophysics from the point of view of physics and physicists.

In what follows, biophysics will be considered with respect to its scope, its relation to other fields of science, its aims and objectives, the training of professional biophysicists, and the relation of biophysics to the teaching of physics.

1. The Scope of Biophysics

The term *biophysics* is used in different connotations by different individuals. It does not represent a conventional scientific discipline, well defined and limited by tradition. It does, to a

TABLE I. Some selected topics in biophysics arranged according to conventional divisions of physics.

(i) Physics of living organisms	(ii) Biological effects of physical agents	(iii) Physical methods for studying biological structures and functions
	I. Molecular, Atomic and Nuclear Physics	
Natural radioactivity in living systems. Particle size and state of aggregation in living systems (crystalloids, colloids, emulsions, gels, etc.). Osmotic phenomena in organisms.	Lethal effects of gamma-rays, alpha- and beta-particles, neutrons, electrons, etc.; effects on growth, regeneration, etc. Effects of isotopes. Effects of polymers. Therapeutic applications of natural and induced radioactivity.	Use of radioactive and nonradioactive isotopes in tracer studies. Use of radioactivity to induce genetic changes.
	II. Mechanics, Wave Motion and Sound	
Stress, strain and strength of materials in relation to the structure and function of organisms. Levers in biological systems. Relation of surface tension, viscosity, etc., to living systems. Hydraulics of blood circulation. Sound production and reception.	Geotropisms. Effects of abnormal gravitational fields. Effects of abnormal pressures. Effects of sound and of ultrasonic vibrations. Responses to changes of stress.	Use of gravitational fields (centrifuge, ultracentrifuge, microscope centrifuge). Manometric and volumetric methods for studying cell and tissue metabolism.
	III. Heat	
Temperature maintenance and regulation in organisms.	Effects of normal environmental temperatures; climatological aspects. Effects of artificially-controlled temperatures. Therapeutic applications.	Calorimetry of living systems. Temperature measurement in organisms and tissues.
	IV. Electricity and Magnetism	
Impedance and other electric characteristics of cells and tissues. Tissue currents, membrane potentials, electric phenomena in nerve conduction, etc. Electric defense mechanisms in organisms.	Effects of atmospheric ionization. Effects of electric and magnetic fields. Effects of electric currents. Therapeutic applications (diathermy, electric cautery, electrosurgery, etc.).	Use of electrical measuring methods and electronic circuits; for example, in measurements of *p*H, oxidation-reduction potentials, nerve action currents, etc. Electron microscopy. Diagnostic applications (electroencephalography, electrocardiography, etc.).
	V. Radiation: (a) Hertzian	
	Production of artificial fevers; other effects on cells and tissues. Therapeutic applications.	Microwave spectroscopy in molecular-structure studies of biochemical interest.
	(b) Infra-red	
Infra-red radiation from living organisms.	Lethal effect on small organisms. Climatological aspects. Therapeutic applications.	Spectroscopy of compounds of biochemica interest. Infra-red microscopy and photography.
	(c) Visible	
Phenomena of vision. Bioluminescence. Photo-sensitization of organisms and tissues.	Phototropisms. Photoperiodism. Photosynthesis.	Use of spectroscopy, polarimetry, refractometry and other optical methods in the study of biochemical compounds. Microscopic methods. Photography, photomicrography, microcinematography, etc.
	(d) Ultraviolet	
Ultraviolet chemiluminescence in organisms; mitogenetic rays (the existence of which is controversial).	Lethal and stimulative actions on cells, tissues and organisms; other effects. Relation to vitamin *D* synthesis, and to health in organisms; climatological aspects. Therapeutic applications. Use in control of microorganisms. Induction of genetic changes.	Spectroscopy of biochemical compounds. Ultraviolet microscopy and microspectrophotometry. Photochemical technics applied to biological problems.
	(e) X-ray (see I *for gamma-rays)*	
	Lethal action on cells and tissues. Effects on growth and regeneration. Therapeutic applications. Induction of genetic changes.	Crystallographic studies of biochemical compounds. X-ray structure studies of cells and tissues. Radiography.

very considerable extent, represent a philosophy of cooperative scientific inquiry by virtue of which physicists and biological scientists join in research to which each can contribute special knowledge and technics. Any investigation that involves the mutual efforts of physicists and biologists is likely, therefore, to be called biophysics, and those who engage in it are likely to be called biophysicists, whether their background is primarily in physics, biology, biochemistry or medicine.

This lack of traditional boundary conditions whereby one can determine with precision what properly is and is not biophysics and who properly are and are not biophysicists has its distinct advantages. It results in a situation in which the fascinating problems of living systems can be investigated freely by inquiring minds with more attention paid to ability and interest than to protocol. In endeavoring to bring the present scope of biophysics into clearer perspective, I have no desire, therefore, to suggest

that the boundaries ought to be fixed and the area of the field made inflexible. My purpose is rather to consider what kinds of inquiry are, by more or less common usage, frequently referred to as biophysical, and to trace the historical trends that have led up to the present status.

From this broad point of view, biophysics may be said to include all applications of physics to the study or explanation of biological systems. Biophysics so defined may conveniently be divided into three aspects, as follows: (i) the physics of living organisms, (ii) the biological effects of physical agents, and (iii) the use of physical methods and measurements in the study of biological structures and functions. Table I summarizes certain phenomena and technics included in this broad field, classifying them according to the traditional subdivisions of physics.

The three aspects of biophysics have their origins in more or less distinct historical trends. These were initiated successively but have developed more or less concurrently.

The first trend had its beginnings in the nineteenth century in the study of the physical aspects of such physiological phenomena as blood flow, temperature maintenance, nerve conduction and respiration. These problems were considered to be a proper part of the domain of physiology. From these beginnings, there developed an enlarging interest in the relation of a variety of physical and physico-chemical properties and phenomena to the structure and function of biological systems. Typical examples are the relation of mechanical stresses, strength of materials, elasticity and viscosity to biological structures, and the relation of osmosis, surface tension, diffusion phenomena and membrane potentials to biological functions.

Such applications of physics to biological problems may be considered to be examples of the physics of anatomy and of physiological processes. This is the classical field of biophysics. Thus, the term *biophysics* appears to have originated with Karl Pearson. In the first edition of his *Grammar of Science*,[1] he proposed that *biophysics* be used to designate "the study of biological phenomena as special examples of physical laws." The Oxford dictionary[2] followed Pearson closely with its definition of biophysics as "the science which applies the laws of physics to explain the phenomena of biology."

It is the physics and physical chemistry of biological systems that has been treated most frequently in texts and monographs on biophysics, as for example those of Burns,[3] Steel,[4] du Noüy,[5] Wishart[6] and Hill.[7] Furthermore, it is such topics that are covered by the biophysics section of English medical examinations in physiology.[8]

The original, or classical, field of biophysics may, therefore, be described as "physical physiology," or, more broadly, "physical biology."

The second trend, which had its beginnings early in the twentieth century and gained special impetus in the 1920's and 1930's, was one toward investigation of the biological effects of physical agents. Applications to clinical medicine had much to do with this trend. Examples of such clinical applications are the use of x-rays and gamma-rays in tumor therapy, the use of sunlight and artificial ultraviolet radiation in the treatment of rickets, lupus and other diseases, and the use of infra-red radiation and high frequency electric fields in the therapy of such diseases as arthritis. More recently, study of the biological effects of physical agents has been stimulated by a variety of medical problems related to special physical factors characteristic of modern conditions of living and working. Examples of such factors are high noise levels (a cause of premature deafness), high atmospheric pressures (a cause of "caisson disease" among divers and caisson

[1] K. Pearson, *The grammar of science* (Adam and Charles Black, 1892), p. 470.

[2] *A new English dictionary, supplement* (Oxford Univ. Press, 1933).

[3] D. Burns, *An introduction to biophysics* (Macmillan, ed. 2, 1929).

[4] M. Steel, *Physical chemistry and biophysics* (Wiley, 1928).

[5] P. L. du Noüy, *Méthodes physiques en biologie et en médicine* (Librairie J. B. Bailliere et Fils, 1933).

[6] G. M. Wishart, *Groundwork of biophysics* (G. Bell and Sons, 1931).

[7] A. V. Hill, *Adventures in biophysics* (Univ. of Pennsylvania Press, 1931).

[8] The following excerpt from the preface to the second edition of Burns (reference 3) is pertinent: "The scope of the book has been slightly altered to make it more in accord with the *Syllabus of Biophysics* suggested by the *General Medical Council*. Sections I and II, with the corresponding exercises in Part II, cover the *Syllabus of The Physical Physiology* required by the *Examining Board in England* of the *Royal College of Physicians* of London and the *Royal College of Surgeons of England*. . . ."

workers), low atmospheric pressures (a cause of physiological disabilities among high altitude fliers) and strong gravitational fields (a cause of special physiological symptoms in dive bombing).

Industrial interest in such problems as the photochemical production of vitamin *D* and the control by lethal ultraviolet radiation of bacterial contamination has played an important role in the support of this field. The pharmaceutical and food processing industries, in particular, have been concerned with numerous problems involving biochemical and biological effects of radiant energy. The biological effects of other physical factors—temperature, humidity and air ionization for example—have evoked industrial and commercial interest, both from the standpoint of the artificial control of physical environments (as in air conditioning) and that of the relation of health to natural physical environments (as in the extremes of temperature and humidity experienced in the tropics).

While practical applications have been important in stimulating interest in this aspect of biophysics, they have not, by any means, been entirely responsible for its development. Thus, those concerned with fundamental biological problems have devoted much effort to (i) the elucidation of certain effects of radiant energy which are of particular biological interest, and (ii) the modification of biological systems or their activities by means of radiant energy as a technic for studying biological phenomena. The first category is exemplified by such phenomena as photosynthesis, photoperiodism and phototropisms. The second is illustrated by the use of radiation to induce genetic changes, a method that has made possible remarkable progress in understanding the role of localized areas (genes) in chromosomes as determiners of the characteristics of developing organisms.

The second trend in biophysics has been concerned, then, with the biological effects of physical agents. Recent developments in nuclear physics have focused attention anew on this aspect of biophysics. The earlier work has been summarized by Duggar,[9] Laurens,[10] Heyroth,[11] Stuhlman,[12] Glasser,[13] and others. The detailed results of more recent research under the Manhattan project, at the University of Chicago and elsewhere, may be expected to become public knowledge as rapidly as declassification and publication permit.

The third trend in the development of biophysics is comparatively new. It has involved the increasing use of physical measurements in the study of the structure and function of organisms. Such technics as electron microscopy, x-ray diffraction analysis, polarization microscopy and stream birefringence, have revealed hitherto unobserved regular structure patterns at the supramolecular level in living systems and their components. Spectroscopic technics, gravitational field methods (for example, the ultracentrifuge) and surface film methods have helped to elucidate the nature of the molecular entities peculiar to biological systems. Methods employing both radioactive and nonradioactive isotopes as tracers have made possible the study of various chemical processes as they take place in living cells and tissues. Refinements in manometric and volumetric technics have permitted studies of gas interchange between cells and their environment, resulting in new and fundamental information concerning metabolic processes.

The third, and most recent, trend in biophysics has been concerned, then, with the use of physical methods and measurements in the study of biological structures and biochemical phenomena. This field has been reviewed by Schmitt,[14] Glasser[13] and Loofbourow,[15] among others.

2. The Relationship of Biophysics to Other Fields of Science

It is evident that many of the phenomena and technics discussed in Sec. 1 and listed in Table I could equally well be considered as indigenous to some other field of science. This arises from the fact that logical choice in the categorical classification of knowledge and methods according to

[9] B. M. Duggar (Ed.), *The biological effects of radiation* (McGraw-Hill, 1936), 2 vols.

[10] H. Laurens, *The physiological effects of radiant energy* (Reinhold, 1933).

[11] F. F. Heyroth, *The chemical action of ultraviolet rays* (Reinhold, 1941).

[12] O. Stuhlman, Jr., *An introduction to biophysics* (Wiley, 1943).

[13] O. Glasser (Ed.), *Medical physics* (Year Book Publishers, 1944).

[14] F. O. Schmitt, "Tissue ultrastructure analysis: polarized light method," reference 13, p. 1586 (Year Book Publishers, 1944); "Ultrastructure and the problem of cellular organization," *Harvey lectures* (series XL, 1944–45).

[15] J. R. Loofbourow, *Rev. Mod. Physics* **12**, 267–358 (1940).

scientific disciplines is affected by emphasis and point of view.

Consider, for example, the study of the molecular weights of proteins by means of the ultracentrifuge. One may class such a study as appropriate to physics if emphasis is upon the general problem of the motion of particles in fluid mediums, as appropriate to physical chemistry if emphasis is upon the use of physical measurements in the characterization of chemical molecules, as appropriate to organic chemistry if emphasis is upon the molecular weights of proteins, and as appropriate to biology if emphasis is upon the relationship of the molecular weights of proteins to their biological functions. Alternatively one may consider that such a study extends all the way from basic physical laws at the one extreme to the characterization of biological systems at the other extreme. Approached from this point of view, and organized so as to include the cooperative efforts of physicists, biologists, organic chemists and physical chemists, such a study is conveniently considered to belong in the borderland category, *biophysics.*

The proper province of any field of science is difficult to define without encroaching on the territory of other fields. In the case of borderland fields this difficulty is especially evident. It seems safe to say, for example, that there is very little so special to biochemistry that it could not be considered, quite logically, as a proper part of physiology, pathology, organic chemistry or physical chemistry. Nevertheless, it is convenient to treat a large group of phenomena under the heading of biochemistry if for no other purpose than to break down the barriers between the parent fields and to achieve a desirable degree of synthesis. The technic of creating a new category in order to secure a certain degree of fusion between old categories is a curious one. Experience has, however, proven it to be effective.

The appropriate relationship of biophysics to the biological and physical sciences would seem, then, to be that of a bridge spanning the gulf between them and affording easy means of passage to and fro. In the interests of keeping traffic moving freely, it would be unfortunate if either physicists or biologists should attempt to preempt the bridge, or to confine the traffic to motion in one direction. If the gulf is eventually filled in to such an extent that the bridge is no longer needed, then it may be time to dismantle the bridge and to move the borders of the biological and physical sciences to some common line near the middle of the gulf. Until that time comes, the bridge serves a useful purpose, though one may expect many differences of opinion as to how long and how broad it should be.

What, then, about the desirable relationship of physicists to biophysics? This question cannot be answered fully until further experience has been gained. The answers to certain aspects of it are, however, clear.

For example, biologists and physicists alike will no doubt agree that there is a pressing need for quantitative observations and measurements in biology, and that the technics of physics are very useful in that regard. It is certainly desirable, therefore, for physicists to cooperate with biologists in making available quantitative technics and methods useful in biological research.

There are, however, many physicists who would be dissatisfied with nothing more than the devising of technics and instruments for use by biologists. Indeed, it is doubtful that such an approach could lead to the most effective exploitation of such technics in all instances. Often one must not only design new physical tools but also use them actively if they are to be conceived and employed most effectively. Is it appropriate, then, that physicists should engage in the active exploration of biological problems?

Certainly there are many biological problems that are sufficiently interesting, complex and challenging to intrigue the most accomplished of physicists, and there is little doubt that combining the study of such problems with the development of methods would often be more stimulating than the development of methods alone. There is, however, one important impediment to the investigation of biological problems by physicists. Whether it can be overcome depends upon individuals and circumstances. This impediment derives from the fact that biological systems are exceedingly complex and varied. Much special knowledge is required to work with them intelligently and to distinguish the significant biological problems from the trivial. The physicist who effectively explores biological problems must, therefore, be competent in the field of

biology as well, or must, at the very least, work closely with capable biologists. If, by such means, it is possible to assure that the biological aspects of the inquiry will rest on firm ground, there would seem to be nothing inappropriate in the active participation of physicists in biological research. Indeed, such a course has often proven highly fruitful in the past, as exemplified by the contributions to biological science of such physicists as Maxwell and Helmholtz.

3. Objectives of Biophysics

In terms of its three aspects, the broad objectives of biophysics are to achieve a better understanding of: (i) the physical properties of organisms and the physical processes occuring in them, (ii) the relationship of organisms to their physical environments, natural or artificial, and (iii) the structure and function of living systems and their components as elucidated by physical measurements.

These objectives may be pursued with practical ends in view, as has been pointed out in Sec. 1. Practical applications occur in such widely diverse fields as public health, clinical medicine, agriculture, food processing, textile manufacture, communication and transportation.

The same objectives may also be pursued in a search for fundamental explanations of those biological phenomena that are at present abstruse. The pooled knowledge and technics of biologists, physicists and biochemists may ultimately lead to the understanding of such processes as reproduction, growth and development in accordance with predetermined patterns, maintenance of dynamic equilibrium in relation to intrinsic and extrinsic factors, transformation of sensory data (intelligence stimuli) into activity (response), and progressive changes in dynamic equilibrium leading eventually to disorganization and disintegration of the living system (death).

Stated in another way, it would seem that biophysics could contribute most to the advance of knowledge if effort were directed primarily toward the following two objectives: (i) the development and use of quantitative methods of observation and analysis for the purpose of accumulating more precise data regarding the descriptive and classificatory aspects of biology, and (ii) the establishment of abstract relationships, or laws, basic to biological phenomena.

The first of these two objectives is important because quantitative observation and measurement are essential preliminaries to the establishment of quantitative generalizations. Furthermore, description and classification can be simplified greatly if it is possible to reduce them to abstract, quantitative, mathematical bases.

Qualitative description now accounts for a very large part of the literature in the biological sciences. This literature is so vast that it is difficult for any one person to become well acquainted with an appreciable part of it. Thus, in the field of plant taxonomy, some 10,000 species of orchids alone have been described. This is approximately one-tenth the number of atomic spectrum lines listed in the *M. I. T. Wavelength Tables*,[16] and yet it represents only a single family of plants. Furthermore, the data for orchids consists in qualitative descriptions, whereas those for spectrum lines include wave-length measurements expressed to from five to eight significant figures.

The description of spectrum lines on a quantitative, systematic basis first involved increased precision in wave-length measurements and the collection of a sufficient amount of quantitative data to permit empirical mathematical relationships to be set up for series in spectra (the Balmer and Rydberg formulas). Such formulas, in turn, indicated gaps in the observations. Many of the gaps were later filled in by experiment. There followed the postulation of atom models, more or less consistent with other physical phenomena (laws of dynamics, of electric attraction and repulsion, and so forth) which would serve to explain the empirical formulas. From this time on, improvements in the observation of spectra, leading to the recording of new lines or to greater precision in the measurement of the wavelength and intensity of previously known lines, went hand in hand with developments in theory from the simple Bohr atom through wave mechanics to modern quantum theory, and predictions from theory as to new lines that might be observed. As a result, both the empirical and

[16] G. R. Harrison, *M. I. T. wavelength tables* (Wiley, 1939).

theoretical aspects of atomic spectra are now well developed.

It is not inconceivable that an analogous continuous "cross-fertilization" of quantitative observation and experimentation on the one hand and theory on the other could be applied to the investigation of a wide variety of biological phenomena, even though such phenomena are characterized, in general, by a far greater number of relevant parameters than the physicist ordinarily has to deal with. Progress along this line has already been made with regard to genetic studies among certain organisms, notably the fruit fly ***Drosophila melanogaster***. Such progress in biology is usually limited, however, by the qualitative nature of the data available.

It is clear that some parameters might easily be measured more quantitatively than has been customary in the past—for example, the color of flower petals, which could be determined more accurately with the aid of physical instrumentation than is now possible by mere visual observations. To obtain quantitative data for certain other parameters might prove exceedingly awkward. But awkward or not, the crux of the problem is to make all the important and relevant observations and measurements more quantitative. This is a direction in which biophysicists might well expend effort profitably.

The comparison made earlier between the classification of orchids and that of line spectra is open to the criticism that the former is concerned with descriptions of gross characteristics of highly organized matter at the macroscopic level, whereas the latter is concerned with the fine structure of matter at the atomic level. The parameters at the higher organizational level are likely to be larger in number and less susceptible of easy quantization. For example, the classification of such familiar objects as chairs in terms of quantized parameters presents considerable difficulty. For that reason, study of the "fine structure" of biological materials at the supramolecular level, by such technics as x-ray diffraction and electron microscopy, may afford better possibilities than study of gross structural characteristics insofar as limiting the parameters and expressing them quantitatively is concerned. The x-ray diffraction pattern of collagen, a protein, has already been shown to vary from species to species. This suggests an approach to comparative histology whereby parameters capable of precise quantitative measurement may be substituted for others that are more vaguely defined.

The second objective, the establishment of abstract relationships and laws basic to biological phenomena, is of course related to the first objective, as has already been indicated. It may be interesting, however, to consider the second objective from the special point of view of the over-all analysis of biological systems.

The activities of living systems may be described grossly as coupled chain reactions involving sequences of metastable dynamic states. Once a living system ceases to involve dynamic processes, it is, in general, dead.[17] The structure of living systems is a kind of dynamically maintained matrix, analogous to the flow form of a stream of water. This has been demonstrated convincingly by isotopic tracer studies.[18]

The dynamic states of living systems are metastable in that their controlling parameters are, in general, modified by the effects of these very same processes. The time continuity of living systems is dependent, in the normal course of events, upon cell growth and cell division—a kind of chain reaction.[19] Many interlocking dynamic processes occur in typical living systems; these are coupled by linkages that involve both time and space parameters.

The analogy of such a system as this to a communications network is apparent. The current in any portion of a communications network is a function of the potential difference applied to the network and of a series of interacting impedances, characteristic of different portions of the system. If the circuit constants and applied potential difference are known, the current in a particular section may be computed by solution of appropriate simultaneous equations. The flow of atomic or molecular entities, characteristic of the dynamic state of a living system, through any one

[17] Some forms of living systems—for example, bacteria—can be maintained at low temperatures, or in a dehydrated state, with little or no metabolic activity without destroying their ability to grow and reproduce when restored to a more favorable environment. During such periods, these organisms are, however, hardly "alive" in the usual sense.

[18] R. Schoenheimer, *The dynamic state of body constituents* (Harvard Univ. Press, 1946).

[19] Living systems, in general, either "die" eventually or divide into two living systems of smaller initial size.

particular part of that system, is analogous to the current in a particular portion of a communications network. The living system is more complicated in that there are many kinds of "currents" in its different parts, and in that the "circuit constants" are variables instead of constants and are modified by the entities that flow in the system. In fact, since living systems, in general, disintegrate after varying periods of time, it would appear that the time variation of "circuit constants" characteristic of any living system in the dynamic, living state is usually such that a considerable number of the "impedances" eventually become infinite, or nearly so.

Obviously the analysis of a living system is fraught with extreme difficulties. Yet if the "currents" could be observed and measured with greater precision, and the "impedance" characteristics determined more quantitatively, there would be hope of developing mathematical theories of living-system function which might, in turn, point to new functions to investigate. Already, with the all-too-crude and largely descriptive data available, Rashevsky[20] and his associates have shown that such an approach can be profitable. Theirs is a beginning; an open road —admittedly rather rough—lies ahead.

These examples serve to illustrate possibilities. They suggest a point of view toward which the efforts of biophysicists might be oriented; a point of view which aims at making the descriptive aspects of biology more precise and quantitative, and at developing relationships and theories that will permit application of abstract laws to biological phenomena.

4. Training of Professional Biophysicists

The advances in knowledge that may be expected to come from biophysical research lie in the direction of the better understanding of biological systems rather than of fundamental contributions to physics. A knowledge of the characteristics of biological systems in general, and a somewhat more thorough knowledge of the special kinds of organisms with which one will be primarily concerned in particular, is therefore essential to the intelligent undertaking of biophysical research. This fact has been mentioned in the discussion of the relationship of physicists to biophysics.

A general background in biology may be obtained from introductory courses in zoology, botany, comparative anatomy, embryology and physiology. These vary in scope, length and content from place to place. Zoology and botany, for example, are often combined in a single course called "general biology." Comparative anatomy may be taught as a part of a general biology course, or it may be a separate subject of instruction. Therefore an appropriate requirement for a general biological background in the training of a professional biophysicist may perhaps best be stated in terms of time: a two- to three-year sequence of lecture and laboratory courses averaging one course per semester and covering the fields mentioned.

The more specialized training in biology best suited to the needs of a biophysicist will depend upon his research interests. It is perhaps not undesirable if some of this training comes rather late, during preparation for the Ph.D. or even afterward. Then it may be approached maturely, with the benefits both of a previously acquired background of methods and of some experience in research. Courses in medical schools, such as histology, biochemistry, physiology and microbiology, may be useful. Such subjects appear to be taught more maturely and thoroughly in medical schools, where more time can be devoted to them and where the average student is older, than in biology departments, though there is sometimes the tendency to orient them more toward medical applications than toward fundamental principles.

So much for biological training; let us consider physics. Following an introductory course in physics, the biophysicist will need, as a minimum, such instruction as is necessary for him to understand physical methods as they are applied to biological problems. These methods include, among others, optical microscopy, polarized-light analysis, x-ray diffraction analysis, electron microscopy, atomic and molecular spectroscopy, isotope tracer technics, ultracentrifuge technics and a variety of methods involving electronic devices. Training should therefore be provided in physical optics, optical instrument design, elec-

[20] N. Rashevsky, *Mathematical biophysics* (Univ. of Chicago Press, 1938); *Advances in applications of mathematical biology* (Univ. of Chicago Press, 1940).

tricity and magnetism, electrical measurements, electronics, atomic structure, atomic and molecular spectra and elementary nuclear physics.

It seems highly desirable that the training in physics go beyond the foregoing minimum. A well-rounded acquaintance with modern physical theory, as embodied in courses on theoretical physics, is useful as training in the analytic methods employed in physics in the study of natural phenomena. More specialized courses, such as theoretical mechanics and theory of electricity and magnetism, are also useful in this regard. Some will wish to go somewhat more deeply into such subjects as x-ray diffraction analysis and crystal structure, spectroscopy and electronics. Time is the essential limitation; with sufficient time available it would be desirable for the professional biophysicist to be trained as broadly and thoroughly in physics as would be the case if he were headed toward a career in physics.

The minimum requirements in mathematics are corollary to those in physics. In addition to advanced algebra, trigonometry and analytic geometry, a working knowledge of elementary and advanced calculus and differential equations is essential, and acquaintance with vector analysis might be desirable. Beyond such basic training, further specialized courses may be useful, depending upon how one's training in physics is oriented.

Finally, training in chemistry is important since the elucidation of many biological phenomena requires a knowledge of molecular structure and molecular reactions. The minimum requirement can be met by the usual courses in general chemistry, qualitative and quantitative analysis and elementary organic chemistry. More advanced training in organic chemistry and in special analytical technics seems desirable.

The desirable backgrounds in these various fields constitute, together, a rather full program to be added to the language and cultural courses and the courses in other special fields of interest which the undergraduate and graduate student will be expected to take. One should consider, therefore, how such a program can be accomplished.

First, there is the question of the proper sequence of courses. Experience seems to show rather clearly that it is more difficult for the average mature person to fill in deficiencies in mathematics and the more quantitative aspects of science than in its descriptive aspects. This implies that training in mathematics and physics should be started in the first college year, and that there should be a continued emphasis on these fields at the undergraduate level, without, however, excluding a somewhat more leisurely approach to the fields of biology and chemistry. In the later undergraduate years, and throughout the graduate period, the emphasis might gradually be shifted to the biological sciences.

Second, there is the question of designation of fields of concentration. If the general sequence just outlined were followed, the undergraduate major field might be physics, and the candidate might then proceed to an advanced degree in biology. How the fields of concentration and degrees are designated would seem to be primarily a departmental and institutional problem. Some institutions may prefer to give advanced degrees in biophysics or physical biology. Some may prefer their candidates to take an extra year or two of training and to proceed to advanced degrees in both physics and biology. The latter policy would be time consuming but might have the advantage of avoiding a situation in which the candidate feels himself to be a sort of hybrid product, without the qualifications of either a physicist or a biologist.

These suggested requirements for working in the field of biophysics are rather formidable. What the requirements ought properly to be is more clear than how they can be satisfied within reasonable time. There are ways of acquiring an approximation to the desirable scope of training through permutations and combinations of standard curriculums. Experience with such permutations and combinations will, in time, help to clarify the issue.

5. Relation of Biophysics to the Teaching of Physics

With increasing interest of biologists in the field of biophysics, it is obvious that it will become more essential for physicists to give consideration to the needs and interests of students in physics whose careers are headed toward biology.

A similar trend took place in the past two decades with regard to students who were oriented toward specialization in chemistry. Interest in the relation of the theory of atomic and molecular structure to chemical problems, and in the use of spectroscopic and other physical methods in chemical research, led to an influx of chemists into physics courses in atomic structure and line spectra, molecular structure and band spectra, x-ray diffraction and crystal structure, nuclear physics and so forth. This migration of chemists into physics courses had an influence on the subject matter and methods of presentation of such courses which becomes apparent if one studies the curriculums of various physics departments over the past 20 years. It is not unreasonable to expect an analogous trend as a result of an influx of biologists into physics courses.

At the elementary level, physics departments of liberal arts colleges are already confronted with the problem of training large numbers of biologists. This arises from the universal requirement of a first course in physics for entrance into medical schools. Attempts to orient elementary physics courses toward the needs and interests of such students have sometimes resulted in an unfortunate tendency to popularize the courses and to teach at a lower intellectual level. This has probably led to some distrust of any effort to consider the special requirements of biology students.

It would seem that the need in elementary liberal arts courses is not to lessen the rigor with which physics is taught, but rather to devote more consideration to the use of examples derived from biological research in illustrating applications of physical principles. Much could probably be accomplished if a considerable number of the problems assigned in an elementary course were drawn from biological and biophysical research. To cite a few examples: problems in centrifugal force could deal with practical examples that arise in ultracentrifugation, problems in calorimetry could be concerned with biological systems and problems involving gas laws could deal with applications of manometric methods employed in biochemical research. It would be possible also to employ supplements to standard physics textbooks in the form of problem books that would outline briefly the biological, biochemical and medical applications of each of the principal fields of physics and would provide a sufficient assortment of well-thought-out problems dealing with these applications to permit the instructor to choose those best suited to the particular needs of his class.

If he is to be able to cite examples drawn from biological research in illustrating physical phenomena, the physics instructor should have a rather broad knowledge of such applications. This implies that instructors in elementary physics courses may need to acquire somewhat broader training than has been customary in the past.

At higher levels of instruction it would seem that course content should, again, reflect cognizance of the relationship of the subject matter to biological phenomena and biological research. In some instances, physics teachers responsible for such courses may develop the required acquaintance with the biological field through research interests that tend toward biophysical problems. Such an effect has been observable in the past in the borderland between physics and chemistry, particularly in courses dealing with atomic and molecular structure and spectroscopy. Other instructors, whose research interests lie elsewhere, may have to make a special effort to acquire some knowledge of the biological and biophysical fields in order to be able to bring a sufficiently broad knowledge to the classroom.

The increasing interest in biophysics, then, offers a challenge to the physics teacher to keep his point of view and knowledge abreast of the times. The need for re-examination of emphasis and expansion of interest seems presently most acute at the elementary course level. This need may be expected to extend to higher course levels as the field of biophysics expands.

Every great advance in science has issued from a new audacity of imagination.—JOHN DEWEY.

Separation of Gases by Single and Double Diffusion

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INTEREST in the method of separating ordinary gases by barrier or single diffusion has greatly increased as a result of the spectacular application of the method to the separation of the uranium isotopes. Very little attention, however, has been paid to the separation of gases by double diffusion. In this latter method, gases are made to flow along both sides of the barrier, the concentration gradient across the barrier being sustained not by maintaining a large pressure difference, as in single diffusion, but by the sweeping action of the second gas.[1] This method is, therefore, different in principle from the separation of gases by single diffusion. It is, in fact, most effective where separation by single diffusion is impossible, that is, where the diameter of the pores in the barrier is many times larger than the mean free path of the diffusing molecules. This fact may be explained with reference to the diffusion cell shown in Fig. 1.

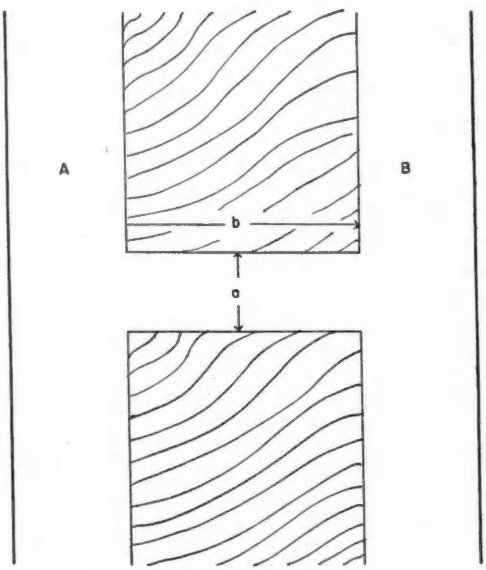

FIG. 1. The diffusion cell.

[1] R. S. Mulliken and W. D. Harkins, *J. Am. Chem. Soc.* **44**, 37 (1922).

Diffusion Cell

In Fig. 1, A and B are two channels connected by a single cylindrical cell or pore of very small cross-sectional area. A pure gas is assumed to flow in A while a vacuum is maintained in B. Under these conditions the current density of the gas passing through the cell from A to B depends only on the pressure in A, the absolute temperature, the molecular weight of the gas, and the ratio of the lengths a and b to the mean free path λ of the diffusing molecules. The three following distinct cases are possible.

Case 1. $a/\lambda \ll 1$, $b/\lambda \ll 1$.—If both a/λ and b/λ are small compared to unity, gas will pass from A to B by the process of *molecular effusion;* that is, the molecules will pass readily through the cell, colliding relatively infrequently with the wall and with each other. In this case i, the current density of the gas expressed in moles per square centimeter, per second, is given by[2]

$$i = P/\sqrt{(2\pi MRT)}. \qquad (1)$$

Here M is the gram molecular weight of the gas, R is the gas constant, T is the absolute temperature, and P is the pressure in A. If a binary mixture is present in A, Eq. (1) will apply to either constituent provided P is regarded as the partial pressure of the constituent in question. If the partial pressure of each constituent is the same, it is evident that the current densities of components *1* and *2* will be related by the equation

$$i_1/i_2 = \sqrt{(M_2/M_1)}. \qquad (2)$$

The problem of making a structurally adequate barrier simultaneously satisfying the conditions $a/\lambda \ll 1$ and $b/\lambda \ll 1$ at a pressure of 1 atmos would be a very formidable one since λ is of the order of 10^{-6} cm at this pressure.

Case 2. $a/\lambda \ll 1$, $b/\lambda \gg 1$.—If b/λ is large and a/λ is small compared to unity, the gas will pass from A to B by *molecular diffusion;* that is, the molecules in passing through the cell will collide frequently with the walls but only rarely

[2] M. Knudsen, *Ann. d. Physik* [4] **28**, 999 (1909).

with one another. In this case, the current density i is given by[3]

$$i=\frac{P}{\sqrt{(2\pi MRT)}}\left(\frac{4a}{3b}\right). \tag{3}$$

Here, again, if a binary mixture is present in A, Eq. (3) will apply to either constituent provided P is regarded as the partial pressure of the constituent in question, and Eq. (2) will give the ratio of the current densities where the partial pressures of the two constituents are the same. This method of separating gases by *molecular diffusion* is frequently referred to as "barrier," or single, diffusion.

Case 3. $a/\lambda\gg 1$, $b/\lambda\gg 1$, $b/\lambda\gg a/\lambda$.—If both a/λ and b/λ are large compared to unity, and b/λ is much larger than a/λ, the gas will be transferred from A to B by *capillary transpiration;* that is, the molecules in passing through the cell will collide very frequently with the walls and *with one another*. For this reason, if a binary mixture is present in A, no separation will be effected.

If now the vacuum in B is replaced by a fast-streaming vapor, the rate of transfer of gas from A to B will be unaffected in cases 1 and 2 because the diffusion will proceed independently in both directions through the cell. It is only necessary that the stream velocities in A and B be large enough to maintain a negligible concentration of invading molecules on both sides of the barrier. The story is very different, however, in case 3, for then the phenomenon is essentially one of gaseous interdiffusion. The simplest situation arises when the pressure in A is equal to that in B. In this case Fick's law of diffusion,

$$\mathbf{i}=-D\operatorname{grad}\rho,$$

may be applied. Here D is the mutual coefficient of diffusion of gas and vapor, and ρ is the gas density. If the density of gas is ρ_0 (mole/cm³) in A and zero in B,

$$i=\frac{\rho_0}{b}D=\frac{P}{RT}\left(\frac{D}{b}\right). \tag{4}$$

Furthermore, if this equation is assumed to apply independently to both components of a binary mixture in A, then the ratio of the current densities will be given by

$$i_1/i_2=D_{13}/D_{23}, \tag{5}$$

provided the partial pressures of the two constituents are the same. In this equation the subscripts 1 and 2 stand for the components of the mixture and the subscript 3 for the vapor.

Let us suppose now that the pressure on the vapor side of the cell is raised slightly above that on the gas side. Then, superimposed on the diffusive flow, there will be a hydrodynamic flow tending to "blow" the diffusing gases backward. Therefore, by properly adjusting the pressure difference across the cell, the slower moving of the two gases may be entirely eliminated, and a small unneutralized current of the faster component will pass into B. The unneutralized component may, of course, be separated from the vapor by condensation.

To give mathematical expression to these ideas, Fick's law of diffusion must be modified by a term that takes into consideration the transfer of gas by convection. The equation of transport then becomes

$$\mathbf{i}=\rho\mathbf{v}-D\operatorname{grad}p.$$

The author has shown[4] that for a pure gas in A and a pure vapor in B this equation yields a current density given by

$$i=\frac{P}{RT}\frac{v}{(1-e^{-vb/D})}, \tag{6}$$

where v is the fluid velocity parallel to the axis of the pore. In deriving this expression the effects of turbulence have been neglected. Also, v has been assumed constant and must therefore be regarded as an average velocity for values of v at which viscous drag plays a role. It will be noted that Eq. (6) reduces to Eq. (4) in the limit as v approaches zero.

If now a binary mixture is present in A, and Eq. (6) is assumed to apply to each constituent independently, the ratio of the current densities will be given by

$$\frac{i_1}{i_2}=\frac{[1-\exp(-vb/D_{23})]}{[1-\exp(-vb/D_{13})]}, \tag{7}$$

[3] M. Knudsen, *Ann. d. Physik* [4] **28**, 75 (1908).

[4] F. A. Schwertz, *Physical Rev.* **68**, 145 (1945).

TABLE I. Values of factor F.*

Second component	F
Methane	1.20
Ammonia	1.28
Carbon monoxide	0.92
Ethylene	1.15
Nitrogen	0.92
Oxygen	0.85
Hydrogen sulfide	1.12
Carbon dioxide	1.01
Sulfur dioxide	1.10

* Data taken from reference 5 except for water, for which diameter of 2.90A was selected.

provided the partial pressures of the two constituents are the same. This expression shows that in double diffusion the ratio of the current densities may be altered by varying two quantities, v and b, which are unrelated to the physical properties of the gases being separated. In barrier diffusion, however, this ratio is shown in Eq. (2) to depend only on the square root of the inverse mass ratio. For *free* gaseous diffusion, that is, where $v=0$, Eq. (5) applies, and the ratio of the current densities is again seen to depend solely on the physical properties of the interdiffusing molecules. In this latter case, both the masses and diameters have roles, since

$$D_{ij}=\frac{3}{2\sqrt{2\pi}}\left(\frac{kT^{3/2}}{P}\right)\left(\frac{1}{\sigma_i+\sigma_j}\right)^2\left(\frac{1}{m_i}+\frac{1}{m_j}\right)^{1/2} \tag{8}$$

to the first approximation, where σ_i and σ_j represent the molecular diameters.[5] Consequently,

$$\frac{D_{13}}{D_{23}}=\left(\frac{m_2}{m_1}\right)^{1/2}\left(\frac{\sigma_2+\sigma_3}{\sigma_1+\sigma_3}\right)^2\left(\frac{m_1+m_3}{m_2+m_3}\right)^{1/2}$$
$$=\left(\frac{m_2}{m_1}\right)^{1/2}F. \tag{9}$$

In Table I the values of the factor F in Eq. (9) are tabulated for binary mixtures of hydrogen with other common gases. Steam is used as the sweeping vapor.

All the values of F are in the neighborhood of unity, indicating that, for the separation of hydrogen from other common gases by *free* double diffusion ($v=0$) into steam, the mass dependence is approximately the same as for barrier diffusion. For *forced* double diffusion ($v\neq 0$), however, the ratio of the current densities, as given by Eq. (7), may be either less or greater than D_{13}/D_{23}, according to whether v is positive or negative. A negative value of v implies that the velocity vector is directed from B to A.

In Fig. 2 values of i_1/i_2 are plotted for the diffusion of H_2 and CO_2 into steam at 150°C and under a total pressure of 1 atmos. The experimental values[6] for the mutual coefficients of diffusion of H_2-H_2O and CO_2-H_2O have been corrected for temperature, giving $D_{13}=1.65$ cm²/sec and $D_{23}=0.32$ cm²/sec. For proper choices of b and v it is evidently possible to obtain complete separation of the two gases, but this result is effected only at the cost of a severe reduction in the magnitude of the current density. Equation (6) has been used to calculate the current density for the diffusion of hydrogen into steam under a pressure of 1 atmos and at 150°C. The results are plotted in Fig. 3. Here the value of i, for $v=0$ and $b=0.01$ cm, is 4.75×10^{-3} mole/cm² sec, corresponding to 107 cm³-of-H_2/cm² sec, or 12,600 ft³-of-H_2/ft² hr, measured at 0°C and under 1 atmos. If hydrogen were merely passed at this rate through a duct of cross-sectional area 1 ft², its linear velocity would be 210 ft/min—a surprising result in view of the common notion that gaseous diffusion is a slow process.

It is also enlightening to calculate the current density to be expected for hydrogen for barrier diffusion under the same conditions, that is, 150°C, 1 atmos, and $b=0.01$ cm. Equation (3) gives, for $a=0.2\lambda=3.4\times 10^{-6}$ cm, 1830 ft³/ft² hr measured at 0°C and 1 atmos.

Inspection of Eq. (3) shows that the magnitude of the current density in single diffusion is inversely proportional to the square root of the absolute temperature and directly proportional to the pressure. For *free* double diffusion, combining Eqs. (4) and (8), it is evident that i is independent of the pressure and directly proportional to the square root of the temperature. Actually, for real gases, D is more nearly proportional to T^2 than to $T^{3/2}$, so that i should be proportional to T rather than $T^{1/2}$. In *forced* double diffusion Eq. (6) makes plain that i is still independent of P since v is inversely pro-

[5] S. Chapman and T. G. Cowling, *The mathematical theory of non-uniform gases* (Macmillan, 1939).

[6] J. H. Arnold, *Ind. Eng. Chem.* 22, 1091 (1930).

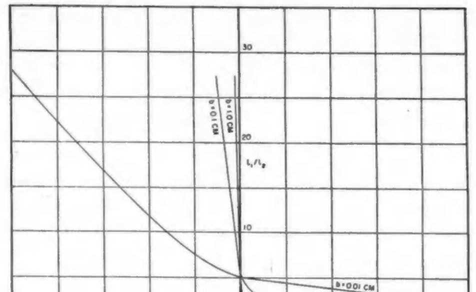

FIG. 2. The relative current density for H_2 and CO_2.

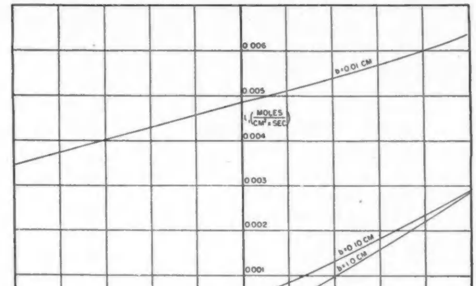

FIG. 3. The current density of H_2.

portional to P. The temperature dependence, however, is complicated by the presence of the exponential term involving D.

The Multicellular Barrier

Any practical diffusion barrier would contain many thousands of cells similar to that illustrated in Fig. 1. In the case of single diffusion, the current density in each cell would be given by Eq. (3). If, therefore, I_1 represents the total flux of species *1*, and I_2 the total flux of species *2*,

$$I_1 = kP_1/\sqrt{M_1},$$

and

$$I_2 = kP_2/\sqrt{M_2}.$$

If now x_1, x_2 and N represent, respectively, the mole fractions of species *1* and *2*, and the total number of moles on the high-pressure side of the barrier, then

$$-\mathrm{d}(x_1N)/\mathrm{d}t = Kx_1N/\sqrt{M_1},$$

and

$$-\mathrm{d}(x_2N)/\mathrm{d}t = Kx_2N/\sqrt{M_2}.$$

The time may be eliminated from these two equations by division, and if the resulting equation is solved, subject to the boundary conditions that $x_1N = x_{1,0}N_0$ when $x_2N = x_{2,0}N_0$, the result is

$$\sqrt{(M_2/M_1)}\ \ln\,(x_2N/x_{2,0}N_0) = \ln\,(x_1N/x_{1,0}N_0).$$

The quantities x_1 and x_2 refer to the gas on the high-pressure, or residual, side of the barrier. If x_1' and x_2' are the corresponding quantities in the diffusate, this equation may be rewritten in the form,

$$\alpha \ln\left(1-\frac{x_2'f}{x_{2,0}}\right) = \ln\left(1-\frac{x_1'f}{x_{1,0}}\right), \tag{10}$$

where

$$\alpha = (M_2/M_1)^{\frac{1}{2}},$$

and f is the fraction of gas permitted to diffuse through the barrier. For very small values of f, Eq. (10) reduces to

$$\alpha(x_2'f/x_{2,0}) = (x_1'f/x_{1,0}),$$

whence

$$\alpha = \frac{x_1'/x_2'}{x_{1,0}/x_{2,0}}.$$

The quantity on the right-hand side of this last equation is commonly referred to as the *separation factor*, and it is apparently equal to the *ideal* separation factor, $\alpha = (M_2/M_1)^{\frac{1}{2}}$, for very small values of f. It is also clear from Eq. (2) that α is the ratio of the current densities when the partial pressures of the two constituents are the same. In other words, α is a measure of the relative diffusibility of the light and heavy components.

In actual single-diffusion experiments, α, as calculated from Eq. (10), will generally be less than $(M_2/M_1)^{\frac{1}{2}}$ because the idealized conditions on which this formula is based cannot be met in practice. The calculated value of α will nevertheless be a measure of the relative diffusibility of the light and heavy components under the actual conditions of the experiment. With this definition of α in mind, Eq. (10) will be applied to some actual double-diffusion experiments.

The calculated values of α will be designated by α_c.

There are very few data in the literature on the separation of gas mixtures by double diffusion. The best information is to be found in a Bureau of Mines Bulletin entitled "Mechanical Concentration of Gases."[7] In it some data are given (page 62) for the separation of hydrogen from nitrogen when a perforated brass sheet was used as a diffusion barrier. The sheet was about 0.016 in. thick and contained 625 perforations per square inch, the diameter of each perforation being about 1/64 in. The sheet was made into a cylinder about 12 cm long and 3.5 cm in outer diameter. The barrier was then placed concentrically in a brass sheath of about 4.8 cm inner diameter. The whole unit was maintained at a temperature of 150°C. Steam was made to flow vertically upward between the sheath and the outer wall of the barrier at a rate of about 3 gm/min, and an H_2-N_2 mixture containing 27.1 percent of H_2 was made to flow vertically downward inside the barrier at a rate of 1.0 l/min, the volume being measured at room temperature and under 1 atmos. By adjusting the flow resistance in the exit line for the purified gas, the pressure drop across the barrier could be altered in such a way that the fraction of feed gas passing through the barrier could be continuously varied. The results are summarized in Fig. 4. The values for the relative diffusibility α_c, calculated by using Eq. (10), are plotted as a function of the fraction f of gas that was permitted to pass through the barrier. The relative diffusibility—that is, the rate at which hydrogen passes through the barrier compared to the rate at which nitrogen passes through it—is seen to decrease as f increases from 0 to 0.4. This result is to be expected since, according to Figs. 2 and 3, large values of i_1/i_2, and therefore large values of α_c, imply small current densities. It is felt that the minimum in the neighborhood of $f=0.4$ is caused by experimental uncertainties, since an error of less than +1 percent in the hydrogen analysis at $f=0.5$ and 0.6 would account for the anomaly. As a matter of fact, α_c probably decreases monotonically.

It would be extremely interesting to plot α_c as a function of the fraction of the steam passing through the barrier, but the necessary data are not available.

The dotted line drawn parallel to the axis of f gives the value of $\alpha_c=\alpha=(M_2/M_1)^{\frac{1}{2}}$ that would be obtained by single diffusion through an ideal barrier, assuming perfect mixing on the high pressure side of the barrier and a perfect vacuum on the other. The data of Fig. 4 are plotted in Fig. 5 in a way that may be more transparent. In Fig. 5 the percentage of hydrogen in that part of the gas passing through the barrier is plotted as a function of f. The upper curve represents the results actually obtained by double diffusion; the lower curve, the results to be expected, under ideal conditions, for single diffusion.

Some Conclusions

The results obtained on the multicellular barrier seem to be in qualitative agreement with the conclusions drawn from the analysis of the unit diffusion cell presented in Fig. 1. Moreover,

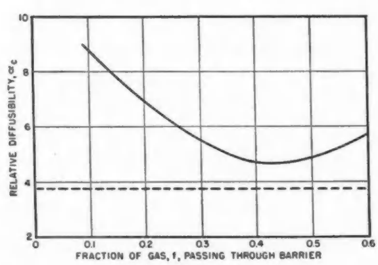

FIG. 4. The relative diffusibility of H_2 and N_2 in single and double diffusion.

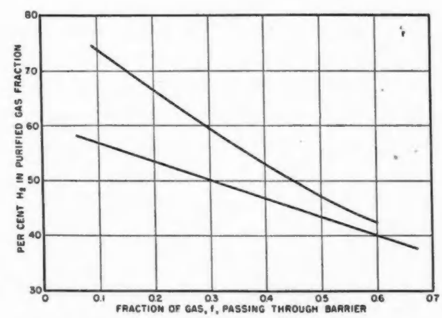

FIG. 5. Comparison of H_2 concentrations in refined fraction for single and double diffusion.

[7] C. G. Maier, *Bureau of Mines Bulletin* No. 431 (1940).

from the experiments summarized in Figs. 4 and 5, it is apparent that the separation factors obtained in actual double-diffusion experiments may be larger than those to be expected from single diffusion under ideal conditions. This outcome is true over a large range of f values, so that, in this respect, the method of double diffusion is superior to single diffusion. Of course, much work remains to be done before a fair evaluation of the relative merits of the two methods may be made.

Little or nothing has been published on the relative speed and economy of operation of the two methods. It is very probable that each method may have its particular advantages for special problems. For example, the separation of heavy isotopes is probably best effected by single diffusion, since the relative diffusibility is given by $(M_2/M_1)^{\frac{1}{2}}$ for single diffusion and, from Eq. (9), by

$$(M_2/M_1)^{\frac{1}{2}}\left[1-\frac{M_2-M_1}{M_2+M_3}\right]^{\frac{1}{2}}$$

for *free* double diffusion. Hence, if $M_2 > M_1$, the sum of the terms in the square bracket is always less than unity. For example, for the separation of $U^{235}F_6$ from $U^{238}F_6$, $(M_2/M_1)^{\frac{1}{2}} = 1.0043$ and

$$(M_2/M_1)^{\frac{1}{2}}\left[1-\frac{M_2-M_1}{M_2+M_3}\right]^{\frac{1}{2}} = 1.0004 \text{ or } 1.0016,$$

depending on whether M_3 is 2 or 200. It is doubtful whether the method of *forced* double diffusion can effect sufficient change in the relative diffusibility of these isotopes to overcome the disparity represented by the foregoing figures because of the vitiating effects of turbulence and viscosity. The picture is more favorable, however, for the separation of light isotopes.

Accidents with Rotating Bodies

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When the good God made the stars as a kind of by-product, he provided forces of gravitation and cohesion which tended to keep such rotating bodies intact. We still do not know the great secret regarding the nature of these forces. Much progress has been made in describing them accurately, but we are quite unable to control or create them at will.

The conditions of failure of rotating bodies have been much studied during the last hundred years, but the subject is still a live one on account of the many industrial applications of rotation and the desirability for many purposes of attaining high speeds of rotation with comparative safety. Some papers have been published this year, but there still seems room for a review of the subject from an historical standpoint. The present attempt may be very incomplete as the literature on the subject is scattered.

The Flywheel

In 1782 James Watt provided his steam engine with a flywheel to make it run smoothly, and he soon realized that if the engine speeded up on account of a change in load the great wheel might tend to fly apart. He was unable at this time to make an accurate study of the strength of a flywheel, for the great development of the mathematical theory of elasticity did not start until about 40 years later, when Poisson, Navier and Cauchy obtained general equations by using theories of molecular forces and general principles. So, Watt took the precaution to add to his engine a governor, which was essentially an improved form of the centrifugal governor invented by Christiaan Huygens for the regulation of a type of clock. The first formula for the strength of a flywheel that I have found men-

* After Professor Bateman's death, on January 21, 1946, there were found among his papers the typescript of this article and a letter of transmittal to the editor, dated August 2, 1945. Apparently the paper and letter had been mislaid and subsequently forgotten.—Editors.

tioned in the available literature was one given by Résal[1] in 1871.

Much interest in the design of flywheels was aroused by a bad accident at Penistone steelworks which was described by Sharp[2] in 1898. In the discussion that followed, Benjamin[3] expressed the opinion that rim joints between arms are bad. He might have mentioned that the stresses in rims and rim joints had been investigated by Lanza[4] in 1895. An old designer, named Fritz,[5] mentioned his continued success with hollow rims and arms that were either hollow or of I-section. Some remarks on the strength of wheel rims were made also by Mansfield.[6] In the following year the French engineer Lecornu[7] gave a mathematical analysis of a type of elastic flywheel that was a generalization of one invented by Raffard in 1890. This flywheel had four satellite masses guided almost radially, while in Lecornu's analysis the masses were supposed to be guided at any angle.

The idea of an elastic flywheel kept recurring whenever there was a bad accident. In 1931, for example, after Ingham[8] had reviewed the causes of flywheel failures with a mathematical discussion and some indication of safety devices, Meissner[9] published an account of a method of possibly reducing the mass by providing oscillating columns of mercury suitably mounted on the wheel. In this connection it may be mentioned that a rubber tube containing mercury was mounted on the rim of the uniform rotation machine devised by Bosanquet[10] in 1883. The use of water in the rim, as suggested by the late Lord Rayleigh, seemed to have little influence in steadying the rotation, especially when the containing pipe was rigid.

Much of the literature on flywheels since 1900 deals with the adaptation of flywheels to electric machinery and engines other than steam engines. Downie[11] discusses the use of flywheels for large low-speed engines for electric lighting and traction; Slichter[12] considers flywheels for engine-driven alternators; Mathot[13] deals with the design of flywheels for large gas engines; Doherty and Franklin[14] describe a type of flywheel for reciprocating machinery connected to synchronous generators or motors. In 1922 Stevenson[15] pointed out that it is wrong to neglect electric forces in calculating flywheels for reciprocating machinery driven by synchronous motors. He then developed a short method of calculating flywheels with special reference to reciprocating ammonia compressors connected directly to synchronous motors. He also gave an account of the historical development of flywheel design from the early efforts of Boucherot to the present day practice of 1925.

Flywheels for steel mill drives were discussed by Umansky[16] in 1923. He remarked that the combination of flywheels and electric motors was then hardly 20 years old and that there were many conflicting and confused ideas about the use of flywheels for electric drives. Low-speed wheels are said to be made usually of cast iron or cast steel, but it is impossible to be certain that a large casting is of uniform structure. Umansky's opinion is that it is not safe to run a large wheel at a rim speed exceeding 8000 to 10,000 ft/min. With a high-speed wheel made of cast steel or boiler plates riveted together, there is more assurance of uniformity of material, and he thinks that rim speeds may safely go as high as 21,000 ft/min, or even higher. Kinkead[17] gives some examples to show the advantage of welded-steel construction for high-speed flywheels. Much, evidently, depends on the choice of suitable material, and a close inspection of material has become common practise. In 1928, for instance, Biedrzycki[18] made a study of the microstructure of some metal in connection with

[1] H. Résal, *Comptes rendus* (Acad. des Sciences, Paris) **73**, 1434 (1871).
[2] A. Sharp, *Engineering* **66**, 357 (1898).
[3] C. H. Benjamin, *Engineering* **67**, 10 (1899).
[4] G. Lanza, *Am. Soc. Mech. Eng.* **16**, 208 (1895); *Engineering* **59**, 72 (1895).
[5] J. Fritz, *Engineering* **67**, 10 (1899); also A. J. Firth, *Engineering* **69**, 80 (1900).
[6] A. K. Mansfield, *Engineering* **67**, 10 (1899).
[7] L. Lecornu, *Comptes rendus* (Acad. des Sciences, Paris) **131**, 253 (1900).
[8] E. Ingham, *Colliery Guardian* **141**, 389 (1930).
[9] E. Meissner, *Schweiz. Bauzeit.* **98**, 15 (1931).
[10] R. H. M. Bosanquet, *Proc. Roy. Soc. London* **34**, 445 (1883).
[11] A. M. Downie, *Engineering* **73**, 98, 134 (1902).
[12] W. L. Slichter, *Am. Soc. Mech. Eng.* **24**, 98 (1903); *Engineering* **75**, 42 (1903).
[13] R. E. Mathot, *Engineering* **80**, 263 (1905).
[14] E. R. Doherty and A. Franklin, *Am. Soc. Mech. Eng. Trans.* **42**, 523 (1920).
[15] A. R. Stevenson, *Gen. Elec. Rev.* **25**, 690 (1922); **28**, 580, 731 (1925); *Refrig. Eng.* **11**, 123 (1924).
[16] L. A. Umansky, *Gen. Elec. Rev.* **26**, 688 (1923).
[17] R. E. Kinkead, *Am. Machinist* **74**, 547 (1931).
[18] R. Biedrzycki, *Tech. Cieplna* **6**, 1 (1928).

the bursting of an $11\frac{1}{2}$-ton 3.1-m flywheel of a Diesel engine.

In 1925 Cutler[19] discussed flywheels for unbalanced air and ammonia compressors for refrigeration, and in 1929 Goss and Putnam[20] gave a history of the development, pointing out that in the case of the ammonia compressor the flywheel can be incorporated in the motor with much success but for the air compressor the problem is different. In 1934 Liwschitz[21] remarked that a reciprocating compressor driven by a synchronous motor generally requires a heavier flywheel than one driven by an asynchronous motor.

The flywheel problem is mathematically somewhat difficult on account of the complex form of the wheel. It is still being discussed. Among recent studies mention may be made of those of Nagashima[22] on the stresses produced by centrifugal force and the stresses produced in the spokes of a wheel by a turning moment applied to the rim. The stresses in the flywheel of an axial blower have been estimated by Schilhansl.[23] Marin[24] bases his estimate of safe speeds for flywheels on a theoretical discussion of stresses in rotating disks. He gives charts for the selection of the allowable speed for a disk of given radius. Ingham[25] has continued his studies of failures of the flywheels of steam engines.

Failure of Grindstones

At this time there seemed to be more concern about the failure of grindstones than the failure of flywheels, and an approximate theory of the stresses in a rotating cylinder was given by John Hopkinson.[26] The approximate theories of the rotating cylinder are classified by Love[27] in his *Theory of Elasticity* into two types. The first, based on the assumption of plane strain, is useful for a discussion of the stresses in a rotating shaft. The second, based on an assumption of plane stress, is applied to the case of a thin disk. In both cases the internal conditions are satisfied but the boundary conditions are only partially fulfilled. On account of the principle of Barré de Saint Venant, the theory is thought to be inadequate only as regards the exact magnitude of certain local stresses, particularly those near surfaces over which the boundary conditions are only partially fulfilled. From the point of view of the modern theory of relaxation of boundary conditions,[28] these approximate theories and the analogous ones used by the designers of disks for steam turbines take a natural position as first approximations.

The Rotating Sphere

In 1885 a French priest, named Rougerie,[29] of Pamiers described to the French Academy of Sciences some experiments with a small artificial terrestrial globe put into rapid rotation in the surrounding air. Something seems to have happened to the reverend scientist, because publication was made only of a preliminary report in which it is claimed that phenomena analogous to the trade winds and other winds known to meteorologists can be observed on a small scale. The globe was said to be provided with "girouettes" to show the direction of the air streams, but there is no description of the method of observation, which was probably stroboscopic. In his paper Rougerie reveals an extensive knowledge of meteorology as then known, and it seems a pity that his work was discontinued.

From an aerodynamical standpoint the problem of the sphere rotating in air is of much interest, particularly when the speed of rotation is so high that the velocity of sound is exceeded at some points of the surface. This is actually the case for the rotating earth, for at the equator the speed is more than 1000 mi/hr. In the case of the earth, however, gravitation helps greatly in keeping the body intact, while with a small

[19] C. W. Cutler, *Refrig. Eng.* **12**, 75 (1925).
[20] H. R. Goss and H. V. Putnam, *Am. Soc. Mech. Eng.* **51**, 117 (1929).
[21] M. Liwschitz, *Elektrotech. u. Maschinenbau* **52**, 159 (1934).
[22] K. Nagashima, *Soc. Mech. Eng., Japan, Trans.* **5**, 24 (1939); *also* Okawa-Chukichi, *Shibaura Rev.* **19**, 297–307 (1940).
[23] M. Schilhansl, *Zeits. f. Ver. deut. Ing.* **84**, 805 (1940).
[24] J. Marin, *Machine Design* **14**, 41, 100 (1942).
[25] E. Ingham, *Eng. & Boiler House Rev.* **55**, 162, 164, 166 (1941).
[26] J. Hopkinson, *Messenger of Math.* (2) **2**, 53 (1871).
[27] A. E. H. Love, *A treatise on the mathematical theory of elasticity* (Univ. Press, Cambridge, 1927), p. 146; (Dover Publications, New York, 1945).
[28] R. V. Southwell, "Relaxation methods in engineering science," *Treatise on approximate computation* (Oxford Univ. Press, New York, 1940).
[29] M. Rougerie, *Comptes rendus*, (Paris) **101**, 568 (1885); *Engineering* **40**, 308 (1885).

globe of ordinary material there is great difficulty in attaining such a high peripheral speed with any degree of safety. If M. Rougerie did try to attain such a speed he may have had an accident.

The Investigations of Charles Chree

Two years after the description of Rougerie's experiments, Charles Chree,[30–35] the well-known geophysicist, began to publish his investigations on the strength of rotating bodies which are now classical and which are still, perhaps, the only completed studies in which the boundary conditions are taken accurately into account. He began with the problem of the solid, homogeneous, isotropic sphere rotating with constant angular velocity Ω about a fixed axis. In the first study gravity was neglected, the surface was supposed to be free from traction and the sphere was not supposed to have an axial hole for the rotating shaft. The chief result was that the greatest stress difference S was given by the formula

$$S=\rho a^2\Omega^2\frac{4\sigma+6}{10\sigma+14},$$

where a is the radius of the sphere and σ is the Poisson ratio. The factor multiplying $\rho a^2\Omega^2$ has values ranging from 0.421 to 0.5, and 17/40 was regarded as a good estimate of its value.

In a later work on rotating spherical shells Chree found values of the factor under consideration for points on the inner surface ranging from 0.422 to 1.082, the first figure being for the case in which the radius b of the inner sphere was zero and the latter when b was $4a/5$. The factor jumped suddenly from 0.422 to 0.831 when b increased from zero to $a/100$. This showed clearly the weakening effect of a small hole and confirmed Hopkinson's inference that a grinding stone would theoretically break with a radial fracture beginning at the inside; but Chree states clearly that he has not proved that the absolutely largest values of either the greatest strain or the maximum stress-difference must occur on the bounding surfaces of the shell, except in the case when the shell is very thin and the afore-mentioned factor is unity.

Though Hopkinson regarded the term "bursting" of a grindstone as mathematically appropriate in the light of his approximate analysis, Chree did not commit himself to any definite opinion. Chree's accurate analysis for the spherical shell shows, moreover, that Hopkinson's statement—"The greater the hole in the center of the stone the stronger will the stone be. A solid stone runs at considerable disadvantage, . . ."—may be somewhat misleading. It must be borne in mind, moreover, that in actual practise the bounding surfaces of a rotating body are not generally free from stress. The shaft on which the body is mounted may have a pinch fit, and the outer surface may carry blades so that effectively there are centrifugal tractions on the outside and a large internal pressure on the inside surface. The boundary conditions are clearly expressed in articles of Martin[36] and Guy[37] on stresses in steam turbine disks.

Problem of the Rotating Cylinder

High speeds of rotation were obtained with cylinders when Charles Wheatstone made his measurements of the velocity of travel of electric disturbances and when Werner Siemens devised an apparatus for measuring small intervals of time for ballistic studies. Rotating mirrors began to be used then for measurements of the velocity of light. Almost immediately after Wheatstone's work of 1835, Arago began some measurements which were continued by Foucault and others. Speeds of 400, 500, 800 and even 1200 rev/sec were used, but there were some accidents that were unexpected and partly inexplicable. Thus Michelson[38] had an accident with a mirror of octagonal section when the speed was only 400 rev/sec. When developing the ultracentrifuge Svedberg had an accident with one of his cells.

An accurate investigation of the problem of the rotating cylinder with flat end surfaces is difficult. Chree, in his investigation,[31] acknowledges that he has been unable fully to satisfy the boundary conditions. This failure may be

[30–35] C. Chree, *Trans. Camb. Phil. Soc.* **14**, 250, 467 (1889); *Quart. J. Math.* **23**, 11–33 (1889); *Proc. Camb. Phil. Soc.* **7**, 201, 283 (1892). *See also* A. E. H. Love, p. 255; A. V. Leon, *Z. Math. Physik* **52**, 164 (1905); **53**, 144 (1906); C. Sunatuni, *Tôhoku Imp. Univ. Tech. Repts.* **9**, 135 (1930).

[36] H. M. Martin, *Engineering* **115**, 1 (1923).

[37] H. L. Guy and P. L. Jones, *Engineering* **115**, 219 (1923).

[38] A. A. Michelson, *Astrophysical J.* **65**, 1–14 (1927).

well understood when note is taken of the fact that some electric problems relating to a finite cylinder are still unsolved. The distribution of electricity in equilibrium on such a conductor is not known accurately, and even the problem of a condenser formed from two coaxial cylindrical plates is still unsolved. The elastic problem of a homogeneous cylinder mounted on a coaxial shaft attached to it by a pinch fit is thus mathematically formidable; it leads in fact to integral equations which might, perhaps, be solved numerically if there were urgent need for an accurate solution.

Problem of the Turbine Disk

There was much interest in rotating systems in the year 1884 when Charles Parsons showed the practicability of the steam turbine. Applications had been made by various men for patents of rotating steam engines, but the Parsons turbine embodied many features that were entirely new. In the investigations that followed, success in obtaining a high speed of rotation was achieved by de Laval whose disk, mounted on a flexible shaft, was rotated at about 30,000 rev/min. The disk was 5 in. in diameter, and so the rim speed was about 650 ft/sec, which is less than the velocity of sound. De Laval apparently made some studies of the strength of thin rotating disks about 1889, when his type of turbine was invented, and a mathematical investigation in the Swedish language was published by Kobb[39] in 1892. In 1889, however, Chree had published his exact solution[31] for the rotating spheroid. The derivation of this solution was simplified by Edwardes[40] in 1895, and an attempt to treat the problem of the disk of uniform thickness by exact methods was made by Chessin[41] in 1905. Exact methods have been used by Chree[32] for the rotating elliptic cylinder of great length when the assumption of plane strain is justified, and this work has been continued by Basu and Sengupta.[42] Mindlin[43] has considered the stresses in a disk rotating about a point other than its center, bipolar coordinates being useful as in previous investigations of Jeffery.[44]

More progress in the study of disks suitable for steam turbines has been made by approximate methods such as those of Aurel Stodola,[45–47] whose writings have added so much to our knowledge of turbines. Tapered disks of various types were studied mathematically. In one important type the thickness z at a distance r from the axis of rotation varies according to a power law, $z=Cr^{-n}$, and in a type designed for uniform stress the law is of type $z=C\exp(-hr^2)$. Since, however, curved disks are hard to make, much attention has been paid to disks formed from portions of conical disks in each of which z and r are connected by a linear law.

This work of Stodola has been carried on by many investigators.[48] A mathematical theory of rotating disks was published by Illeck[49] in 1906. In Austria, also, Basch and Leon[50] extended the theory of the Stodola-Laval disk of equal resistance to centrifugal force in a work of 1907. It may be recalled that, on the assumption that the problem is one of plane stress, the radial stress R and the tangential stress T (or hoop stress) in Laval's economical type of solid disk were made constant by the adoption of a profile of type $z=C\exp(-hr^2)$. Basch and Leon found under the same assumption those sets of solutions giving a constant value to only one of the principal stresses, say T, and a smaller value to the other one R. They found also profiles for which one of the principal extensions had a constant value while the other one had a smaller value. The search for profiles with given distributions of stress has been continued by Arrowsmith[51] and, more recently, by Grammel,[52] who has added a new type of profile to the list of

[39] G. Kobb, *Stockholm Öfversigt* (1892), pp. 571–574.
[40] D. Edwardes, *Quart. J. Math.* **27**, 81 (1895).
[41] A. Chessin, *Bull. Am. Math. Soc.* **12**, 58 (1905).
[42] N. M. Basu and H. M. Sengupta, *Calcutta Math. Soc.* **18**, 141 (1927); *see also* S. Ghosh, *ibid.*, **25**, 99 (1933).
[43] R. D. Mindlin, *Phil. Mag.* (7) **26**, 713 (1938).

[44] G. B. Jeffery, *Trans. Roy. Soc. London* **221A**, 265 (1920).
[45–47] A. Stodola, *Steam and gas turbines* (McGraw-Hill, 1927), Vol. 1, p. 376; *Engineering* **86**, 440–445 (1908); *Z. Ver. deut. Ing.* (1907), p. 1269, *Eng. Abstracts* (1907), p. 291. *See also* S. H. Weaver, *Gen. Elec. Rev.* **20**, 791–799 (1917).
[48] J. Prescott, *Applied elasticity* (London, 1924), pp. 343–346.
[49] J. Illeck, *Österreichischen Ing. Architekt Ver.* (1906); *Eng. Abstracts* (1907), p. 291.
[50] A. Basch and A. Leon, *Vienna Acad. Sci.* **116**, 1353 (1907).
[51] G. Arrowsmith, *Engineering* **116**, 417 (1923).
[52] R. Grammel, *Ingenieur Archiv* **7**, 137 (1936).

ready-made profiles listed in books such as that of Malkin.[53]

It should be mentioned that approximations based on the method of W. Ritz have been used for some of these profiles, but most writers prefer to use the approximate theory in which a differential equation of the second order is used to derive the distribution of stress for a disk with given profile. In the case of a law of type $z=Cr^{-n}$, this equation has simple solutions and gives the formulas of Stodola,

$$R=D[(3+\sigma)ar^2+(\alpha+\sigma)b_1r^{\alpha-1}+(\beta+\sigma)b_2r^{\beta-1}],$$
$$T=D[(1+3\sigma)ar^2+(1+\alpha\sigma)b_1r^{\alpha-1}+(1+\beta\sigma)b_2r^{\beta-1}],$$

where $D=E/(1-\sigma^2)$, σ is the Poisson ratio and E is Young's modulus. The indices α and β are given by the equations

$$2\alpha=n+(n^2+4\eta\sigma+4)^{\frac{1}{2}},\quad 2\beta=n-(n^2+4\eta\sigma+4)^{\frac{1}{2}}.$$

These formulas are applicable to a disk with central hole. Charts to facilitate their use were prepared by Martin[54] in 1912 and by Knight[55] in 1917. Holzer reduced the formulas to a concise form by introducing the stresses at some prescribed point, and this device was adopted by Arrowsmith[51] in his calculations of 1923.

In 1923 Martin published some numerical calculations for a disk of conical type for which the linear differential equation of the second order is hypergeometric. As in the paper of 1912 the differential equation is obtained by a principle of least work done by the radial stress R and the hoop stress T, both of these stresses being assumed to be functions of R only. The differential equation is then the same as that obtained by the method of Stodola. The principle may be compared with Holzer's rule for constructing a disk of minimum material when the aim is to make the work of deformation, under otherwise equal circumstances, as high as possible.[56]

The hypergeometric equations occurring in Martin's analysis have integral exponent difference, and so one solution is of logarithmic type near one of the singular points, the other solution is of logarithmic type near the other singular point. The power series which represent solutions thus converge slowly near the singular points. Despite this disadvantage Martin[57] and later Honegger[58] constructed some tables of the functions and obtained a few numerical results. These were recognized by them to be inadequate, and last year Bisshopp,[59] of the Fairbanks Morse Company, constructed more complete tables by using both the logarithmic solutions and the power series for computations. The properties of the logarithmic solutions were developed in more detail than usual, and calculations were made for a type of disk needed for a supercharger.

It may be of interest to recall that in the work of Suhara and Yaskawa[60] done in 1929 the profile was determined by a law of type

$$z=z_0(1-br)^k,$$

with $k=\frac{1}{3}, \frac{1}{2}, 1, 2, 3$. For an integral value of k the exponent difference for the resulting hypergeometric equation was again integral, but some numerical calculations were made.

Many other investigations of stresses in rotating disks might be described. The method of Moyes[61] is an extension of that of Arrowsmith[51] and is worked out for a Curtiss disk with a neck radius of 8.48 in. and a speed of 6000 rev/min. The hoop stress at the hub is estimated at about 21,780 lb/in.². In the second paper in which reference is made to the work of Donath, described by Stodola,[46] a method that can be easily applied is developed in which much use is made of mean stresses.

Some new solutions of the problem of the rotating disk were given by Gran Olsson[62] in 1937. In 1939 a method of calculation suitable for disks of any symmetrical cross section and

[53] I. Malkin, *Festigkeitsberechnung rotierender Scheiben* (Berlin, 1935).

[54] H. M. Martin, *Engineering* **94**, 279 (1912); *Design and construction of steam turbines* (London, 1918).

[55] W. Knight, *Engineering* **104**, 109, 310 (1917).

[56] J. Holzer, *Zeits. f. ges. Turbinenwesen* 401 (1913), 4 (1915); *see also* A. Stodola, *Steam and gas turbines*, Vol. 1, p. 395.

[57] H. M. Martin, *Engineering* **115**, 1–3, 115–116, 407–630 (1923).

[58] E. Honegger, *Zeits. f. angew. Math. Mech.* **7**, 120–128 (1927); *see also* the book of Malkin.

[59] K. E. Bisshopp, *Am. Soc. Mech. Eng., J. Applied Mech.* **11**, A1–A9 (1944).

[60] T. Suhara and H. Yaskawa, *Proc. Imp. Acad. Tokyo* **5**, 72–74 (1929).

[61] S. J. E. Moyes, *Engineering* **142**, 109–111 (1936); **147**, 241–243 (1939).

[62] R. Gran Olsson, *Ingenieur Archiv* **8**, 270–275, 373–381 (1937).

external loading was given by Gruber,[63] and in the same year Yaskawa[64] gave a method for calculating the stresses produced by centrifugal force in a disk with a sudden change of thickness as r increases from hub to tip.

The problem of revolving plates with holes not in the center was discussed to some extent in the book of Stodola,[46] but a further discussion has been given recently by Schultz-Grunow.[65]

A new method of calculating stresses in rotating disks has been developed recently by Wegner.[66] It is claimed that this method, which seems to be of a general character, overcomes some of the disadvantages that are encountered when use is made of the method of approximation developed by W. Ritz for the solution of the boundary problems of mathematical physics.

In a still more recent investigation Lake[67] has given a simplified method of determining hoop stresses in fan rotors and has compared the approximate method with more exact methods. The new approximate method is based on the assumption that the rotor section displaces radially under stress without distortion. It is intended for a rotor with central hole and fan blades attached to the outer boundary. Calculations were made for a side-wheel plate rotor of an ammonia works compressor fan.

Comparisons between Theory and Experiment

Some tests have been made that do not wholly confirm the approximate theories but, as Stodola and others have pointed out, the tests were made under conditions in which the basic assumptions of the approximate theories were not fulfilled. In the experiments of Brown and Fitzgerald[68] made in 1907, tests were made of a solid rubber disk 12 in. in diameter, $2\frac{1}{2}$ in. thick in the middle and $\frac{1}{2}$ in. thick at the edge. This disk was of straight or conical profile. A second disk tapered according to a hyperbolic curve $z=c/r$ had a thickness of 3 in. at the center and a thickness of $\frac{5}{8}$ in. at the tip. There was also a central hole $1\frac{1}{2}$ in. in radius. The opinion was expressed that Stodola's formulas for the calculation of tapered disks are unreliable, but in the first place Stodola's formulas were derived on the hypothesis of small strains and with the rubber disks used in the experiments the extension amounted to from 24 to 30 percent of the original dimension of the disk. Also, Stodola assumed a variation of thickness so small that the angle formed by the directions of stress acting on a coaxial cylindrical section of the disk and the plane of symmetry could be neglected. This assumption was inapplicable to a disk with a 1-to-3 slope of one face. Stodola doubted also whether the assumption of constant moduli of elasticity would be valid in the case of rubber. However, Fitzgerald, in answer to Lanchester's inquiry, pointed out that up to an extension as high as 35 percent the stress-strain curve for the rubber used was found to be nearly linear.

In the experiments of Pollock and Collie,[69] in which conical disks were tested to destruction, the angle between two faces at the fast moving rim of the disk was as high as 46°. The observed breaking speed was often not much more than half the estimated breaking speed. Indeed, in one case, with a disk 11.1 cm in diameter, the estimated breaking speed was 2420 rev/sec, and the observed breaking speed only 800 rev/sec. Two ways of estimating the breaking speed were indicated, but the approximations used may not have been applicable on account of the greatness of the slope of the sides of the disk. These experiments were made in connection with the design of an ultracentrifuge for which the rotating element must have a certain thickness to make it suitable for the desired experimental work. The disks were made of forged aluminum alloy treated in various ways.

A graphical method of determining stresses in various types of rotors for high-speed centrifuges has been developed by Pickels.[70] It is pointed out that the failure of a rotor to rupture during a few test runs at a roughly estimated maximum

[63] W. Gruber, *Forsch. Gebiete Ing.* **10**, 1–129 (1939).
[64] H. Yaskawa, *Soc. Mech. Eng. Japan* **5**, I, 153–154, 154–159 (1939).
[65] F. Schultz-Grunow, *Zeits. f. angew. Math. Mech.* **16**, 345–377 (1936).
[66] U. Wegner, *Zeits. f. Ver. deut. Ing.* **87**, 443–444 (1943); *Forsch. Gebiete Ing.* **13**, 144–149 (1942).
[67] G. F. Lake, *Am. Soc. Mech. Eng., J. Applied Mech.* **12**, A65–A68 (1945).
[68] J. Brown and M. F. Fitzgerald, *Engineering* **86**, 371–372, 386, 510–511 (1908); *see also* H. H. Jeffcott, p. 440.
[69] H. C. Pollock and C. H. Collie, *Engineer* **163**, 102–103 (1937).
[70] E. G. Pickels, *Rev. Sci. Instruments* **13**, 101–114, 426–434 (1942).

speed is not a reliable criterion of safety. Material may become fatigued, and rupture occur without any obvious warning. Hence the need for a careful mathematical study. The choice of a suitable material is also a matter of great importance. In the design of turbine rotors the rule that the material must be the very best for the purpose has been followed strictly for some time and a careful plan of inspection devised.

Tests which seem to confirm Stodola's approximate theory were made by Samuelson[71] in 1913. In this case also, the conditions of the theory were not well fulfilled. Figures were given for the expansion at the hub at speeds ranging from 1000 to 3750 rev/min, and a limit was set for the forced fit of a steel disk on a steel shaft in order that the disk may not become loose on the shaft at a speed of 3000 rev/min. Good agreement between experimental and calculated values is claimed also by Yaskawa[64] with an agar-agar model and a reasonable value of Poisson's ratio suggested by Yamaguti. A photoelastic study of the stresses in a rotating disk has been made also by Newton.[72]

Stodola did not remain content with his approximate solution, for in 1907 he made a second approximation[47] in which the nonuniformity of the stresses and strains over a cylindrical section of the disk is taken into consideration. Though the solution found was not exact, it agreed very well in the case of a spheroid with the exact solution obtained by Chree.[31]

In the actual problem of the steam turbine disk, temperature stresses have to be taken into consideration when there is a change of condition. These stresses are discussed in the book of Stodola[46] and in a recent paper by Beck.[73] When the rim is much hotter than the hub the temperature stresses augment the stresses due to centrifugal force and the pull of the blades.

The effect of axial restraint on the stress in a rotating disk has been considered by Green.[74]

[*Editors' note*—One page was missing from the typescript sent to us and was not found in a subsequent search of Professor Bateman's papers. Evidently it contained the references to footnotes 75 to 83, inclusive.]

[71] F. Samuelson, *Engineering* **95**, 621 (1913).
[72] R. E. Newton, *Am. Soc. Mech. Eng., J. Applied Mech.* **7**, A57–A60 (1940).
[73] E. Beck, *Forsch. Gebiete Ing.* **10**, 164–169 (1939); also E. Honegger, *Brown Boveri Rev.* **16**, 243–253 (1929).
[74] W. G. Green, *Phil. Mag.* (7) **1**, 1236–1251 (1926).

Attainment of Very High Speeds of Rotation

By eliminating the supporting shaft Henriot and Hugenard[84] were able 20 years ago to obtain very high speeds of rotation. Critical speeds of sympathetic vibration of rotor and shaft were avoided altogether, and the rotor, being freed from any solid or liquid contact, could choose its own axis of rotation. The stator consisted of a conical nozzle with vertical axis, while the lower part of the rotor was also more or less conical but with grooves to enable the whirling air produced by jets to get a grip. The rotor was then sucked towards the stator until a position of equilibrium was attained, the phenomenon being much like that discovered many years ago at the iron works of Fourchambaut and investigated by Desormes,[85] Willis[86] and others. This work has been carried on by Girard and Chukri[87] who have developed a centrifuge of perfect stability and great angular velocity.

In the more recent work of MacHattie[88] a ball of diameter $\frac{3}{32}$ in. was spun up to 110,000 rev/sec. The peripheral speed was thus nearly 2600 ft/sec, more than double the velocity of sound. Even at this high speed the deformation of the rotor may be small. It may be shown from Chree's results that in order that the axial drop of a point on the surface of a sphere and on the axis of rotation may be as much as one-tenth of the radius, the peripheral speed at the equator must exceed the velocity of propagation of transverse waves in the material and this may be very high.

It should be mentioned that the way in which a sphere is deformed by centrifugal forces de-

[75–79] H. Lamb and R. V. Southwell, *Proc. Roy. Soc. London* **99A**, 272–280 (1921), **101**, 133–153 (1922); *see also* M. Koenig, *Rugby Eng. Soc. Proc.* **21**, 37–66 (1926–7); F. Dubois, *Schweiz. Bauzeit.* **89**, 149–153 (1927); P. G. Hensel, *Power Plant Eng.* **46**, 62–63 (1942); I. Malkin, *J. Franklin Inst.* **234**, 355–369, 431–452 (1942).
[80] A. Stodola, *Steam and gas turbines*, Vol. 1, Sec. 187.
[81] J. M. Downer, *Gen. Elec. Rev.* **29**, 829–832 (1926).
[82] W. Campbell, *Gen. Elec. Rev.* **27**, 352 (1924) and references given by Malkin in ref. 79.
[83] A. Meyer and H. B. Saldin, *Am. Soc. Mech. Eng., J. Applied Mech.* **9**, A59–A64 (1942).
[84] E. Henriot and E. Huguenard, *Comptes rendus* (Paris) **180**, 1389 (1925).
[85] C. Desormes, *Ann. chim. physique* **36**, 69 (1827); Hachette, *ibid.*, **35**, 34, 53 (1827).
[86] R. Willis, *Trans. Camb. Phil. Soc.* **3**, 129 (1828).
[87] P. Girard and Ch. Chukri, *Comptes rendus* (Paris) **196**, 327 (1923); *also* P. Girard and N. Marinesco, *ibid.* **206**, 2000 (1938).
[88] L. E. MacHattie, *Rev. Sci. Instruments* **12**, 429 (1941)

pends very much on the value of the Poisson ratio. This is emphasized by Sunatani.[85]

Appendix 1

In the work of Chree and Edwardes on the spheroid of revolution the components of displacement (u, v, w) are given by expressions of type $u=xQ$, $v=yQ$, $w=zQ'$, where Q and Q' are linear functions of z^2 and $r^2=x^2+y^2$. The dilatation $\Delta[\equiv\partial u/\partial x+\partial v/\partial y+\partial w/\partial z]$ is also a linear function of z^2 and r^2. If $m=\lambda+\mu$, $n=\mu$, where λ and μ are Lamé's constants for the substance; Q, Q' have the forms

$$Q=r^2\left(3A''+\frac{m+2n}{m}A'-\frac{\rho\Omega^2}{8m+8n}\right)-2z^2\left(3A''+\frac{6m+4n}{m}A'\right)+B''-\frac{m+2n}{m}B',$$

$$Q'=-2r^2\left(2A'+\frac{6m+3n}{m}A''\right)+4z^2\left(2A'+\frac{m+n}{m}A''\right)+2B'-\frac{2m+2n}{m}B'',$$

while Δ has the form

$$\Delta=-(2n/m)r^2[3A''-4A'+\tfrac{1}{4}\rho\Omega^2m/n(m+n)]+2z^2(4A'-3A'')+2B'+B''.$$

The constants A', A'', B', B'' are determined by the boundary conditions at the free surface,

$$(r^2/a^2)+(z^2/c^2)=1,$$

and are actually found by solving the equations

$$(7m+2n)A'+(6m+2n)A''-(3mB'-nB'')/a^2=(2m+n)H,$$

$$[20mc^2+(16m+2n)a^2]A'+[6nc^2+(18m+6n)a^2]A''+3mB'-nB''=0,$$

$$(16m+8n)A'+(18m+6n)A''+[2nB'-(3m+n)B'']/c^2=0,$$

$$[(16m+8n)c^2+4na^2]A'+[(18m+6n)c^2+(15m+15n)a^2]A''-2nB'+(3m+n)B''=2(n-m)a^2H,$$

where $H=\rho\Omega^2m/8n(m+n)$.

In the case of the sphere, $c=a$ and there is a simplification,

$$Q=\frac{1}{5m+5n}\left[\frac{5m+n}{3m-n}a^2-r^2-z^2\right]+\frac{(4m-n)a^2+m(r^2-2z^2)-\frac{1}{2}(5m-n)(r^2+z^2)}{n(19m-5n)},$$

$$Q'=\frac{1}{5m+5n}\left[\frac{5m+n}{3m-n}a^2-r^2-z^2\right]+\frac{m(r^2-2z^2)+(5m-n)(r^2+z^2)-(8m-2n)a^2}{n(19m-5n)}.$$

The quantity $xu+yv+rw$ changes sign when

$$\frac{1}{5m+5n}\left[\frac{5m+n}{3m-n}a^2-r^2-z^2\right](r^2+z^2)+\frac{(8m-2n)a^2-(3m-n)(r^2+z^2)}{2n(19m-5n)}=0.$$

At the surface of the sphere, where $r^2+z^2=a^2$, the points where the radial displacement vanishes lie on the cone

$$\frac{4(r^2+z^2)}{15m-5n}+\frac{(5m-n)(r^2-2z^2)}{n(19m-5n)}=0.$$

Within the cone points move inwards, while outside the cone points move outwards. The angle of the cone depends appreciably upon the value of the Poisson ratio σ, which is connected with m and n by the equation

$$1+2\sigma=m/n.$$

According to Newton the materials commonly used for rotors have values of σ ranging from $\frac{1}{4}$ to $\frac{1}{3}$, but for the Bakelite used in the method of "freezing stresses" the value is $\frac{1}{2}$ at 230°F.

Appendix 2

The mean radial stress R and mean hoop stress T are connected by the equations[48]

$$rzT=r(\mathrm{d}/\mathrm{d}r)(rzR)+\rho\Omega^2r^3z,\quad rR-\sigma rT=r(\mathrm{d}/\mathrm{d}r)(rT-\sigma rR),$$

of which the first is the force equation and the second a condition of compatibility. With $r=e^s$, the first equation may be satisfied by writing

$$rzR+\rho\Omega^2\int ze^{3s}\mathrm{d}s=F,\quad rzT=F',$$

where primes denote differentiations with respect to s. The second equation may be satisfied identically by writing

$$r(R-\sigma T)=G',\quad r(T-\sigma R)=G.$$

The differential equation for G is

$$zG''+z'G'+(\sigma z'-z)G+\rho\Omega^2(1-\sigma^2)ze^{3s}=0,$$

and this is equivalent to the equation for ξ given by Stodola.[89] Indeed, if ξ is the mean radial displacement, $G=E\xi$, where E is Young's modulus. The differential equation for F is

$$0=zF''-z'F'+(\sigma z'-z)F+\rho\Omega^2\left[z\int ze^{3s}\mathrm{d}s+\sigma z^2e^{3s}-\sigma z\int ze^{3s}\mathrm{d}s\right],$$

and this is equivalent to the equation given by Prescott.

In these equations $2z$ denotes the thickness of the disk at radius r, ρ is the density, supposed uniform throughout, and Ω is the angular velocity of rotation.

[89] Reference 80, p. 375.

What is most needed for learning is an humble mind.—CONFUCIUS.

Some Applications of the Laplace Transform

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1. IN recent years operational methods have been introduced to simplify many physical and engineering problems. One of the most useful is the application of the Laplace transform.

For intermediate courses in mechanics and electricity, as offered by departments of physics in colleges and universities, the student has completed a year of general physics, differential and integral calculus, and perhaps a course in elementary differential equations. In many cases this last course is taken concurrently. Further, most modern intermediate textbooks employ mathematical methods beyond the scope of such students. It is difficult, therefore, for the student of average mathematical ability to grasp the material presented in these intermediate courses. The use of the Laplace transform tends to overcome this difficulty.

The transform enables the student to utilize boundary conditions with much more ease than the methods of differential equations. Thus a realization of the physical significance of these conditions is more readily obtained.

2. Given a function $\mathrm{F}(t)$, defined for $t>0$, the operation

$$\int_0^\infty e^{-st}\mathrm{F}(t)\mathrm{d}t=\mathrm{f}(s) \tag{1}$$

on $\mathrm{F}(t)$ is called the Laplace transformation of $\mathrm{F}(t)$. The function $\mathrm{f}(s)$ is the Laplace transform of $\mathrm{F}(t)$. The operation (1) defines a correspondence between the functions $\mathrm{F}(t)$ and $\mathrm{f}(s)$. For example, t and $1/s^2$ are corresponding functions. Tables of transforms, inverse transforms and many theorems on convergence, existence, limits and so forth, which the use of the transform requires, are found in standard reference books.[1–4]

It is not necessary for the student to perform the integration (1) for each problem. The tables referred to[5] may be used in just the same way as tables of integrals, sines, cosines, logarithms, exponentials and the like. If the student were required to compute these functions, the method itself would be worthless because the computation would take more time than the classical methods of handling these problems, and would be more involved. To illustrate this, let

$$\mathrm{f}(s)=\frac{1}{s^{\frac{1}{2}}(s-a^2)}.$$

Using the tables, we find that[6]

$$\mathrm{F}(t)=(1/a)e^{a^2t}\,\mathrm{erf}(a\sqrt{t}).$$

This would be a rather complicated calculation for the undergraduate. Such results, however, are readily available for use.

The purpose of this paper is to acquaint the instructor with the technic of the Laplace transform for the solution of elementary physical problems. By judicious use of illustrative problems, clearly indicating the method and use of tables, the normal course work may be greatly simplified. The method will be outlined for a simple mechanical problem, and then developed for other physical topics.

3. *Velocity of a body moving through air with resisting force approximately proportional to the velocity.*—Assume $v=v_0$ when $t=0$. From Newton's second law, if m is the mass of the body, and a a constant, $a>0$,

$$m\mathrm{d}v/\mathrm{d}t=-av,$$

or

$$\frac{\mathrm{d}v}{\mathrm{d}t}+\frac{a}{m}v=0. \tag{2}$$

If $\mathrm{f}(s)$ is the transform of v, Eq. (2) becomes[7]

$$s\mathrm{f}(s)-\mathrm{F}(+0)+\frac{a}{m}\mathrm{f}(s)=0.$$

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[1] R. V. Churchill, *Modern operational mathematics in engineering* (McGraw-Hill, 1944).
[2] H. S. Carslaw and J. C. Jaeger, *Operational methods in applied mathematics* (Oxford Univ. Press, 1941).
[3] G. Doetsch, *Theorie und Anwendung der Laplace-Transformation* (Springer, Berlin, 1937).
[4] D. V. Widder, *The Laplace transform* (Princeton Univ. Press, 1941).

[5] Such as are found in Churchill, reference 1, pp. 295–302.
[6] Reference 1, No. 40, p. 297.
[7] $L\{\mathrm{F}'(t)\}=s\mathrm{f}(s)-\mathrm{F}(+0)$, reference 1, No. 3, p. 294; $\mathrm{F}(+0)$ denotes the limit of $\mathrm{F}(t)$ as t approaches zero through positive values.

$$\mathrm{f}(s)=\frac{\mathrm{F}(+0)}{s+(a/m)}=\frac{v_0}{s+(a/m)}.$$

The inverse transform is

$$v=v_0e^{-(a/m)t},$$

which is the solution of the problem.

4. *Oscillations of two coupled pendulums, connected as a simple one-dimensional coupled oscillator.*—Two pendulums of lengths l_1 and l_2, and masses m_1 and m_2, respectively, are suspended a fixed distance apart from the same horizontal support, and are connected by a spring, parallel to the support, and at a fixed distance below it. The coupling coefficient for the first pendulum alone is k_1; for the second pendulum alone, k_2; and for the spring, k_3. The general equations for the oscillator may be written

$$\begin{aligned} m_1\frac{d^2x_1}{dt^2}+(k_1+k_3)x_1-k_3x_2&=0,\\ m_2\frac{d^2x_2}{dt^2}+(k_2+k_3)x_2-k_3x_1&=0. \end{aligned}\tag{3}$$

Equations (3) may be rewritten in terms of frequencies:

$$\begin{aligned} \frac{d^2x_1}{dt^2}+\omega_1^2x_1-\frac{k_3}{m_1}x_2&=0,\\ \frac{d^2x_2}{dt^2}+\omega_2^2x_2-\frac{k_3}{m_2}x_1&=0, \end{aligned}\tag{4}$$

where $(k_1+k_3)/m_1=\omega_1^2$ (first pendulum oscillating, second at rest, $x_2=0$); $(k_2+k_3)/m_2=\omega_2^2$ (second pendulum oscillating, first at rest, $x_1=0$); $k_3^2/m_1m_2=\omega_{12}^2$ (coupling term). Applying the Laplace transform to Eqs. (4), we obtain

$$\begin{aligned} &s^2\mathrm{f}(s)-s\mathrm{F}(+0)-\mathrm{F}'(+0)\\ &\qquad+\omega_1^2\mathrm{f}(s)-\frac{k_3}{m_1}\mathrm{g}(s)=0,\\ &s^2\mathrm{g}(s)-s\mathrm{G}(+0)-\mathrm{G}'(+0)\\ &\qquad+\omega_2^2\mathrm{g}(s)-\frac{k_3}{m_2}\mathrm{f}(s)=0, \end{aligned}\tag{5}$$

where $\mathrm{f}(s)$ is the transform for x_1, $\mathrm{g}(s)$ that for[8] x_2.

The general solution contains the functions $\mathrm{F}(+0)$, $\mathrm{F}'(+0)$, $\mathrm{G}(+0)$ and $\mathrm{G}'(+0)$ and will be of the form

$$\mathrm{f}(s)=\frac{s^3\mathrm{F}(+0)+s^2\mathrm{F}'(+0)+s\{\mathrm{G}(+0)(k_3/m_1)+\omega_2^2\mathrm{F}(+0)\}+\mathrm{G}'(+0)(k_3/m_1)+\mathrm{F}'(+0)\omega_2^2}{(s^2+\omega_1^2)(s^2+\omega_2^2)-\omega_{12}^2}.\tag{6}$$

The usual boundary conditions imposed on this system are

$$\begin{aligned} \mathrm{F}(+0)=1,\quad \mathrm{G}(+0)=0,\\ \mathrm{F}'(+0)=0,\quad \mathrm{G}'(+0)=0. \end{aligned}\tag{7}$$

When the boundary conditions (7) are applied, the general solution (6) becomes

$$\mathrm{f}(s)=\frac{s^3}{(s^2+a^2)(s^2+b^2)}+\frac{\omega_2^2s}{(s^2+a^2)(s^2+b^2)},\tag{8}$$

where a^2 and b^2 contain ω_1^2, ω_2^2 and ω_{12}^2. The values of a and b are the two resultant frequencies of oscillation for the system. Applying the inverse transform to Eq. (8), one obtains

$$\begin{aligned} x_1&=\frac{a^2\cos at-b^2\cos bt}{a^2-b^2}+\omega_2^2\frac{\cos bt-\cos at}{a^2-b^2},\\ &\qquad a^2\neq b^2,\\ x_1&=C_1\cos at+C_2\cos bt. \end{aligned}\tag{9}$$

In like manner,

$$\begin{aligned} \mathrm{g}(s)&=\frac{m_1(s^2+\omega_1^2)}{k_3}\left\{\frac{s^3}{(s^2+a^2)(s^2+b^2)}\right.\\ &\qquad\left.+\frac{\omega_2^2s}{(s^2+a^2)(s^2+b^2)}\right\}-\frac{m_1s}{k_3},\\ \mathrm{g}(s)&=\frac{m_1\omega_{12}^2}{k_3}\frac{s}{(s^2+a^2)(s^2+b^2)},\\ x_2&=\frac{m_1\omega_{12}^2}{k_3}\frac{\cos at-\cos bt}{b^2-a^2},\quad a^2\neq b^2,\\ x_2&=C_3\cos at+C_4\cos bt. \end{aligned}\tag{10}$$

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[8] $L\{\mathrm{F}^n(t)\}=s^n\mathrm{f}(s)-s^{n-1}\mathrm{F}(+0)-s^{n-2}\mathrm{F}'(+0)-\cdots-\mathrm{F}^{n-1}(+0)$, reference 1, No. 4, p. 294. As before, $\mathrm{F}(+0)$ represents the limit of $\mathrm{F}(t)$ as t approaches zero through positive values; $\mathrm{F}'(+0)$ is the time derivative under the same conditions; similar expressions hold for $\mathrm{G}(t)$.

This method of solution has the advantages of not using the characteristic equation for the solution of Eqs. (3), and of obtaining the characteristic or proper frequencies without the direct solution of a quartic. However, if numerical values of a and b are wanted, the solution of a quartic whose coefficients contain ω_1^2, ω_2^2 and ω_{12}^2 is necessary. The general solution given in Eqs. (10) is adequate for elementary work. The characteristic frequencies, along with ω_1^2, ω_2^2 and ω_{12}^2, may be represented on the Mohr circle diagram of two-dimensional elasticity.[9]

Another solution of the same problem may be made with Lagrangians. If the angles with respect to the vertical for each pendulum are used as variables, the kinetic and potential energies are readily obtained. There results a pair of simultaneous equations in terms of the angles, identical in form with Eqs. (3). These may be solved by using similar boundary conditions. The angle solution gives the same pair of proper frequencies.

5. *Harmonic vibration of a beam, simply supported at the ends.*—For a beam of uniform cross section parallel to the yz plane, the normal deflection $w(x,t)$ is measured downward if the x axis is taken along the axis of the beam. The basic equation is

$$EI\frac{d^4w}{dx^4}-m\omega^2w=0, \tag{11}$$

where E is Young's modulus; I, the moment of inertia of the cross section with respect to the y axis; m, the mass per unit length; and ω, the angular frequency. Rewriting Eq. (11), using the abbreviation $\alpha^4\equiv m\omega^2/EI$, one obtains

$$\frac{d^4w}{dx^4}-\alpha^4w=0.$$

Application of the Laplace transform gives

$$s^4f(s)-s^3F(+0)-s^2F'(+0)\\ -sF''(+0)-F'''(+0)-\alpha^4f(s)=0.$$

The appropriate boundary conditions are

$$F(+0)=0,\quad F(+l)=0,$$
$$F''(+0)=0,\quad F''(+l)=0.$$

[9] J. P. den Hartog, *Mechanical vibrations* (McGraw-Hill, 1934).

Hence

$$f(s)=\frac{s^2F'(+0)+F'''(+0)}{s^4-\alpha^4}.$$

The inverse transform is

$$w=\frac{F'(+0)}{2\alpha}\{\sinh\alpha x+\sin\alpha x\}\\ +\frac{F'''(+0)}{2\alpha^3}\{\sinh\alpha x-\sin\alpha x\},$$

$$w=A_1\sinh\alpha x+A_2\sin\alpha x.$$

At $x=l$,

$$A_1\sinh\alpha l+A_2\sin\alpha l=0,$$
$$A_1\sinh\alpha l-A_2\sin\alpha l=0.$$

These are satisfied if $A_1=A_2=0$, or $\sinh\alpha l\sin\alpha l=0$. This gives $\alpha l=n\pi$, for integral values of n. Hence $A_1=0$, and A_2 is undetermined. The resulting vibrations are

$$w_n=A_n\sin(n\pi x/l),$$

and the frequencies are

$$\omega_n=\frac{\pi^2n^2}{l^2}\sqrt{\left(\frac{EI}{m}\right)}.$$

Here $n=1$ represents the fundamental vibration; $n=2$, the first harmonic, and so on.[10,11]

6. *Two series R, L, C inductively coupled circuits, freely oscillating.*—For this case there is no applied electromotive force in either circuit; hence the equations are

$$L_1\frac{d^2q_1}{dt^2}+R_1\frac{dq_1}{dt}+\frac{q_1}{C_1}=-M\frac{d^2q_2}{dt^2},$$

$$L_2\frac{d^2q_2}{dt^2}+R_2\frac{dq_2}{dt}+\frac{q_2}{C_2}=-M\frac{d^2q_1}{dt^2},$$

where the subscripts 1 and 2 apply to the two separate circuits. The boundary conditions may be chosen so that

$$q_1(0)=q_0,\quad q_1'(0)=i_0,$$
$$q_2(0)=0,\quad q_2'(0)=0.$$

Application of the Laplace transform and the

[10] T. von Kármán and M. A. Biot, *Mathematical methods in engineering* (McGraw-Hill, 1940).

[11] I. S. Sokolnikoff and R. D. Specht, *Mathematical theory of elasticity* (McGraw-Hill, 1946).

boundary conditions yields

$$L_1\{s^2q_1(s)-q_0s-i_0\}+R_1\{sq_1(s)-q_0\}+\frac{1}{C_1}q_1(s)=-Ms^2q_2(s),$$

$$L_2s^2q_2(s)+R_2sq_2(s)+\frac{1}{C_2}q_2(s)=-M\{s^2q_1(s)-q_0s-i_0\}.$$

These are of the same form as Eqs. (5). Solving for $q_1(s)$ and simplifying, one obtains

$$q_1(s)=\frac{c_1s^3+c_2s^2+c_3s+c_4}{s^4+\alpha s^3+\beta s^2+\gamma s+\delta},$$

where the eight constants are expressible in terms of the respective values of R, L and C, and of M, the mutual inductance between the circuits.

The general solution of this transform depends, naturally, on the numerical values of the circuit constants. The quartic of the denominator may be solved by the usual methods of the theory of equations and broken down into four linear factors; thus,

$$q_1(s)=\frac{c_1s^3+c_2s^2+c_3s+c_4}{(s-r_1)(s-r_2)(s-r_3)(s-r_4)}=\frac{\phi_1(s)}{\phi_2(s)}.$$

The form of the inverse transform depends on the nature of the four roots $r_1, \cdots, r_4$. Use of Sturm's theorem and related analytic methods is helpful. If the r's are real and distinct, the inverse transform is

$$q_1(t)=\sum_{j=1}^{4}\frac{\phi_1(r_j)}{\phi_2'(r_j)}e^{r_jt}.$$

Similar expressions are found for $q_2(t)$. The solutions for charge may be transformed into solutions for currents. These solutions represent two harmonic, damped oscillations in each circuit, producing a periodic rise and fall of amplitude superposed on a steady falling off due to damping. This process is similar to the production of acoustic beats between two musical notes.

7. These illustrations show the applicability of the method of the Laplace transform. Many others which may be introduced into intermediate mechanics and electricity courses are given in the references.[1, 2, 11, 12]

There are limitations to the widespread use of the method. Certain transforms and their inverses must be expressed in terms of complex variables. The use of these would introduce undesirable complications in the intermediate work. The method is applicable to partial differential equations and to Sturm-Liouville systems, as treated in advanced mechanics.

[12] L. A. Pipes, "Application of operational calculus to the theory of structures," *J. Applied Physics* **14**, 486 (1943).

HISTORY shows that great economic and social forces flow like a tide over communities only half conscious of that which is befalling them. Wise statesmen foresee what time is thus bringing, and try to shape institutions and mold men's thoughts and purposes in accordance with the change that is silently coming on. The unwise are those who bring nothing constructive to the process, and who greatly imperil the future of mankind by leaving great questions to be fought out between ignorant change on one hand and ignorant opposition to change on the other.—John Stuart Mill.

Modernizing the Undergraduate Physics Curriculum: Proposed Change at Washington University

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WE have just finished an extensive reorganization of the curriculum in the physics department at Washington University. It will go into effect in the fall of 1946–1947. This article is a discussion of the new curriculum. The old curriculum has been in effect for well over 15 years. It is exceedingly simple, and its description occupies less space in the catalog than that of almost any other field. The reason for this is that it would have been foolish to have offered an elaborate variety of courses when the average number of undergraduate students taking all courses beyond an introductory course seldom exceeded 12 or so.

We have three types of course in beginning physics: one for regular college students, one for premedical students, and one for engineering students. Their scope may be indicated by the fact that the textbook for the first two courses is Stewart's *Physics* and that for the engineering courses is Hausman and Slack's *Physics*. Because of the tightness of the engineering curriculum, engineering students normally do not go any further in physics. College students needing more physics would in our program go on to *Modern physics* and then to *Advanced general physics*. These three courses formed the core of our undergraduate program. Somewhat off the main line we have listed a course in x-ray manipulations, usually taken by premedical students, and a course in electrical measurements, usually taken by students of electrical engineering. These few courses have in the past constituted our whole offering for undergraduate majors in physics. A more elaborate program could not be justified when so few students elect to take intermediate and advanced undergraduate courses.

To offset the fewness of undergraduate courses, we have always encouraged able seniors, and occasionally juniors, to take selected graduate courses when they have finished our undergraduate offerings.

We are now at a turning point in physics at Washington University, as is true, of course, everywhere.

Physics, though it is the most fundamental of all the sciences, has been less known to the general public than chemistry, botany, geology and other sciences. We have all been amused, and at times somewhat annoyed, by the stories of the draft boards that could not find it in their hearts to regard physics as a science; and of personnel officers in some large companies—whose names are household words—who have listed physicists in their organizations as chemists or engineers but not as physicists. Thanks to the outstanding contribution of physicists to the winning of the war, the public is now thoroughly familiar with the words *physics* and *physicist*. The public has also heard repercussions of the statement that there is a hugh deficit in physicists that will take ten years or more to make up. All this means that many people are becoming interested in the possibility of physics as a profession, and many more would like to learn something about physics even though they do not plan to make a livelihood at it. It is, therefore, inevitable that in all universities we must expect a large increase in the number of students taking physics at all levels. At Washington University we shall have in 1946–47 about double the number of instructors we had before the war, and, in addition, we are organizing an Institute of Nuclear Science and Engineering which will naturally be closely related to the physics department.

In common with other physics departments we are constantly receiving requests to supply physicists to industrial laboratories—more requests than we can possibly fill. Up to the war such requests almost invariably were for men who had been trained to the Ph.D. level. I notice, however, an increasing interest among the representatives of industrial laboratories in men who have been trained to the Bachelor's level and to the Master's level as well as in men who have the Ph.D. I, therefore, no longer find it necessary to tell the student who asks about the outlook for employ-

ment as a physicist that it is virtually useless to count on getting a job unless he has a doctor's degree. Finally, this greatly increased awareness of physics is bound to divert more students into physics in secondary schools, and this in turn will call for more teachers of physics. All these various considerations lead me to the conclusion that right now we are fully justified in expecting and encouraging many more students to major in physics at both the undergraduate and the graduate level than was the case before the war.

In attempting to plan a curriculum, one has necessarily to work within the framework of certain boundary conditions. These may be internal or external. By internal boundary conditions I mean those determined by general university and college policy. These may vary considerably from one institution to another. By external boundary conditions I mean those conditions that have nothing to do with any particular university but are determined by the nature of the subject itself and by human nature.

At Washington University a student wishing to major in physics must enrol in the College of Liberal Arts. We do not have a physics curriculum in the School of Engineering. In the College there are certain prescribed regulations within which any curriculum must fit. A student must take two years (12 semester hours) in English, in social sciences and in a foreign language. He must also take 6 semester hours in classical culture. These requirements take 42 units out of the 120 required for graduation, leaving 78 units for everything else. By the time a student finishes his sophomore year he must elect an area of concentration which would naturally center around his major subject. We have a rule that not over 36 units can be taken in any one department. We feel that a student majoring in physics should take a course in mathematics each year, totaling about 24 units. It would be desirable for him to have at least 12 units in chemistry. These units in physics, mathematics and chemistry add up to 72. We have only 78 units available after the broad requirements of the College have been satisfied, leaving very little for free electives.

Many universities provide for a physics curriculum within the School of Engineering. There are certain advantages in this, especially for students aiming at industrial physics. It has to be admitted that many good physicists are deficient in their knowledge of large scale applications; for instance, the characteristics and use of generators, motors and transformers and the precautions that have to be taken to operate them safely. Few graduate students know much about handling apparatus using hundreds of amperes of current although they are quite familiar with the instruments using minute currents. A physics curriculum in a School of Engineering could and should include enough instruction in mechanical and electrical engineering to fill this and other gaps in their preparation.

Should our activities expand greatly and should we not be able to provide for our needs within the framework of the College rules, we may well request permission to offer a program in physics in the School of Engineering. It is not my thought that such a program would have the same relation to physics as chemical engineering has to chemistry. It would still be physics as we now know it. It may well be that the training of physicists in our new Institute of Nuclear Science and Engineering would fit in more naturally within the framework of the School of Engineering than within the College of Liberal Arts. We could undoubtedly give a more concentrated course in physics and associated subjects than is now possible within the College, through the omission of certain required courses outside the area of concentration and the removal of the 36-unit maximum in the major field.

Let us now consider what I have called the external boundary conditions, those conditions that are not peculiar to any one university. The volume of important fundamental basic physics has increased almost explosively within the lifetime of all of us. A hasty survey of the elementary textbooks in use in 1905 and in 1945 indicates very clearly the extraordinary number of new ideas, concepts, processes and instruments that writers now feel impelled to discuss, or at least to mention briefly. On the other hand, the capacity of students to learn and to comprehend has undergone no appreciable improvement in the same period. It is just not realistic to assume that present-day students can absorb substantially more in a four-year period than students in our day or in the days of our parents could.

The dilemma to be resolved, therefore, is how to deal with a situation in which we have a much larger volume of knowledge to be handled but no greater capacity in which to put it. To my mind it is futile to add course upon course to take care

of the new material that has accumulated. A way to meet the situation is to exercise a rigorous selection in the material we offer in our courses, to decide what can well be omitted, so as to make place for topics that have become highly significant and important, and, wherever possible, to seek principles that are common to two or more fields and save time by teaching them just once instead of each time we meet them. It is only by rigorously selecting material and, as my colleague, Professor Jauncey, aptly describes it, by collapsing it so that one operation of learning will cover more than one field of application, that we can hope to do something towards partially meeting the situation. Perhaps we can console ourselves by reflecting on the problem that will face our successors, the physics teachers of 1996. I am well aware of the validity of the viewpoint that repetition is often an essential element in learning, that many seeds must be planted for every one that becomes a growing plant. Nevertheless, it seems to me that the amount of repetition that we habitually give can be cut down drastically with advantage. Students have come to count unconsciously on the availability of repetition and so, if we are going to reduce it in amount, it is only fair to the student to get him conditioned to the change in the situation.

Since the volume of fundamental physics has grown so much, it could be argued that the proper thing to do is to extend the undergraduate period from four to five, or even to six years. Doubtless the same line of thought could be used to extend the time devoted to the Ph.D. curriculum. This is an easy solution, but I do not believe that it is a good one. Frankly, I do not want physics to get into the same state as medicine, in which a student first puts in four years to get a bachelor's degree and then four more years to get an M.D., and after that two to three years as a resident or interne. The end result is that a physician seldom begins to earn his living or practice his profession until he is over thirty. We all know that, in physics, much research of really first-class quality has been turned out by men well under thirty. During the war we have seen many men who were in the middle twenties in various war projects undertake difficult problems and carry them through rapidly and intelligently to a successful conclusion. In view of such considerations I am opposed to a program that stretches out the learning period and so defers the age at which a man can begin original work or use his physics professionally. I believe that a man need not necessarily know an enormous amount of physics before he tries his hand at research or uses his knowledge in some other way. Too much knowledge can have a discouraging effect on some people. I well remember J. J. Thomson stating his opinion that a certain degree of healthy ignorance was an asset to a research worker.

The crux of the problem of planning an undergraduate curriculum in physics, as I see it, is to decide what to drop in order to make room for what we now consider significant and important. That it can be done and has been done one knows from experience. For instance, as an undergraduate student I learned in the beginning course, and in the intermediate course which followed it, a lot about geometric optics, including the construction and characteristics of various eyepieces, the various aberrations, how to combine two lenses to reduce chromatic aberration, how to estimate the residual chromatic aberration and what could be done about that; the various kinds of telescopes and microscopes, rainbows, haloes, caustics, thick lenses and so forth. All these topics were treated quantitatively though, to be sure, in a simple way. If someone had said in 1910, let's drop 80 percent of this material from the curriculum, I am sure that many good but conservative instructors would have felt that a valuable discipline was being needlessly torn away. Yet most of this material does not now appear in any undergraduate curriculum and we get along happily without it. If occasionally we have a problem involving these displaced topics we solve it by writing to Bausch and Lomb, or Zeiss, and ask what they can supply to meet our needs.

Again, those of us who took advanced undergraduate physics 30 or 40 years ago will no doubt remember that all kinds of synthetic problems in mechanics were accepted features of physics courses. No one worries today because such "disciplines" have either been relegated to the background or drastically curtailed. Our problem today, as I see it, is to find a way of deciding what to omit to make room for topics that are today

fundamental and significant. Our choice will be right if 30 to 40 years from now the omitted topics will not have been reinstated. Unfortunately, we cannot wait for this test; we have to act as best we can.

One cannot adequately discuss the nature of, and the reasons for, changes in the curriculum in physics at the junior and senior levels without discussing the setting in which they have been made. Hence these preliminaries.

All of the junior and senior courses have as prerequisite a six-unit course in *Modern physics*, the scope of which is best indicated by the textbook of that name by Professor Jauncey, who gives the course. Originally this course was planned for a specific purpose, to provide for students who could take only one course in physics beyond the beginning course. It was planned to give them a broad view of the interesting parts of modern physics and in as quantitative a way as seemed possible on a foundation consisting of beginning physics and one year of college mathematics. In this way our view was sharply differentiated from the view that underlies survey courses based on no prerequisite at all. Although the course still fulfills its original functions, it turns out also to be an excellent prerequisite for students who know that they definitely want to go on in physics. For such students it provides a stimulating insight into a variety of the interesting modern fields which they can look forward to studying again at a higher level. We are, therefore, retaining this course as the standard prerequisite for all junior and senior courses, all of which are on the same level in the sense that no one of them is a prerequisite for any other. Students will normally take, with *Modern physics* a course entitled *Intermediate laboratory*, which meets once a week for two semesters. In this, they will carry out selected experiments at a higher level of difficulty than in the elementary laboratory.

In our new program we offer our junior and senior students a choice of five one-semester courses, each with three units of credit, that is, 45 one-hour lectures. There is no laboratory work associated with the individual courses. This may seem a defect in our plan. Had physics a unique and privileged position in the university and had we complete control over a student's program, we could no doubt organize a better program for him from the point of view of teaching him physics. However, we recognize the limitations that exist and consequently try to avoid, as far as possible, setting up a program which could cause us difficulties in applying it to occasional individual cases. We offer *Advanced laboratory*, which is required of students majoring in physics, with four units of credit, or two three-hour meetings per week throughout the year.

A student majoring in physics would be required to take the *Advanced laboratory*, and courses in electricity, nuclear physics, optics of light, x-rays and electrons; and would be strongly urged, or possibly required, to take the courses in heat and thermodynamics and in the physics of solids. Counting the preliminary work, he would take courses in physics totaling somewhere between 27 and 36 units. We shall require him to take four years of mathematics, amounting to about 24 units; two of these years should be in courses beyond the introductory course in calculus. He will be advised, but not required, to take from 10 to 20 units in chemistry. As an elective, a course in electrical engineering could be recommended.

It is important, however, that we should always be on guard against the almost irresistible tendency to add one required course on top of another until the student is heavily loaded down. It is vitally important that the student should have ample time to reflect and to study. I also favor a plan whereby a student should be required to study certain areas of physics by himself and be tested in his mastery of them by examination. However, we have no provision for this at present.

In planning these advanced undergraduate courses, let me discuss the types of student we have in mind. First, there is the student who is studying physics because he proposes to use it professionally; in general, he will go into graduate work, but may very well seek a job after obtaining his bachelor's degree. Second, there is the student who is taking physics as a minor to some other field such as mathematics, chemistry or electrical engineering. Finally, there is the student who is majoring in physics because he likes it, but who does not intend to use it in earning his living.

I think that it would be taking too narrow a

view of our function to plan our courses exclusively on the assumption that all our students are heading for the doctorate and intend to become professional physicists. We should plan all courses so that a wider circle of students can attend them profitably. This, of course, is difficult, but it is a problem that must be attacked if we are to succeed in interesting a larger group of students in our science.

I shall now describe the six courses that we propose to offer as our advanced undergraduate curriculum in physics.

Nuclear Physics

No excuse is needed today for introducing a course in nuclear physics at the advanced undergraduate level. No field in physics has developed at such a rapid pace in the last decade or two. As is often the case when a new field is opened up, the basic ideas, or at any rate most of them, are easy for the undergraduate student to grasp. Nuclear physics covers an enormous range, from purely theoretical ideas to intensely practical aspects. Our problem is to decide how best to present in the time available—45 lectures—those aspects of the subject that will give to undergraduate students the best over-all picture of its status today.

Since the course on *Modern physics* is a prerequisite, and since it contains a section on radioactivity, we have a clear situation to which we can apply our principle of not repeating what has once been given. The instructor in *Nuclear physics* should not go over the same ground again as though it had never been covered before, and so needlessly use up several of his 45 lectures. He could very well require the students to review what they have had, and possibly check the adequacy of the review by a brief test.

The course affords a good opportunity for presenting and developing some very fundamental principles. Thus collision problems could be worked out mainly to account for the scattering of alpha-particles and the collision between neutrons and protons, but the instructor should not fail to point out that the basic principles apply, with little change, to astronomy and to collisions between billiard balls. Nuclear collisions also afford a fine opportunity for emphasizing the principles of conservation of energy and of momentum and of mass-energy equivalence. The behavior of beta-particles serves to introduce the student to the need for replacing Newtonian mechanics by relativity mechanics. Simple statistical theory could be worked out from first principles to account for the random distribution in time in the emission of alpha-particles. This is but a particular case of the statistical theory, and the instructor would do well to bring home the idea that the theory can be applied to a variety of phenomena in totally different regions of physics and, indeed, outside physics altogether.

With such a wealth of material available, it is a real problem to decide what fraction of it to present as representing best the fundamentals of this field. The available textbooks were written before the war, and anyhow none of them seems quite suitable for use in this course with the type of student we expect to find taking it. Probably the instructor will find it necessary to mimeograph his own notes. If I were asked which of the available text books could be taken as a guide as to the scope and emphasis that should be developed in this course, my answer would be that *Radioactivity and Nuclear Physics* by Cork and the chapters on nuclear physics in the book by Stranathan and in that by Richtmyer and Kennard come nearest.

Electricity

The course in electricity has to be highly selective, for we have the formidable problem of trying to compress into 45 lectures a reasonably satisfactory condensation of the essentials that can be absorbed at the advanced undergraduate level. We propose to curtail drastically the amount of so-called classical electrical theory that we give. By classical electrical theory I mean the subject substantially as it is presented in Jeans, which, to be sure, is too difficult for an undergraduate course, or in Starling, or in Page and Adams. Those who worked through the problems in, say, the book by Jeans certainly acquired a dexterity in solving them, but, after all, too many of the problems were of a synthetic type and far removed from actual problems that come up experimentally. I believe that the approach to electricity found in Harnwell's book would be a better one to follow, though without doubt that book covers more than one could

reasonably expect an advanced undergraduate to master in a one-semester course. I believe that there should be an introduction to the behavior of nonlinear elements in circuits, since they are becoming increasingly important today. In the classical approach only the linear elements were recognized. My colleague, Professor Jauncey, believes that it would be of value to develop the idea that linear elements should be thought of as limiting cases of nonlinear elements.

The course would naturally include a pretty careful handling of alternating-current phenomena under sinoidal impressed emf. Complex quantities should be used extensively. It is becoming more and more necessary to acquaint the student with the methods of handling the mathematics of the application of pulses to networks. This calls for a simplified introduction of Heaviside's approach, or a modern equivalent. We have found a small book by Carter, on electrical transients,[1] to be an excellent and compact introduction to this field.

There is a small book by Olson, called *Dynamical analogies*,[2] that develops side by side the solutions of analogous problems in electricity, sound and mechanics. It embodies the principle we hope to emphasize in our approach in all our courses, namely, that wherever we work out a basic procedure we shall apply it to widely different fields and thus avoid loss of time through needless repetition. I do not mean that I propose to have Olson's book used as a text; I merely cite it as an excellent illustration of the process of collapsing knowledge.

It will be noticed that we are not contemplating a separate course in electronics. Our undergraduate students will have to get the elements of electronics in this course on electricity or get a more extended course in the electrical engineering department.

This course on electricity, as I envisage it in outline, will constitute quite a departure from the conventional course on the subject. To work it out satisfactorily is a challenge, but I believe that it will be done.

Light, X-Rays and Electrons

We now turn to the course that is to take the place of the conventional course on physical optics. I am not sure what is the best title to use. I shall tentatively call it *Light, x-rays and electrons.*

Ordinarily the course on physical optics is limited, at any rate implicitly, to the properties of radiations between wavelengths of, say, 8000 and 3000A. However, the principles developed apply over a vastly greater range. The same fundamental theory of interference applies equally well to x-rays and microwaves as to visible light. The same is true of reflection, polarization, dispersion and wave propagation. Why, then, not develop at one and the same time the basic principles that are common to all these fields? The student will gain by the continued insistence on what is common to widely different and apparently wholly unrelated fields.

Another excellent illustration of the procedure is the discussion of wave and group velocities. One could develop the theory quantitatively and illustrate its application in light and in water waves. One could certainly mention its significance in wave mechanics, but that is about all that one can do, for at this stage wave mechanics is a field which our students have not entered.

It is easy to develop the fundamental principles common to electron optics and to light, though of course the practical working out of electron lens systems can be carried out more simply by procedures that are not suitable for working out glass lens systems.

Heat and Thermodynamics

Our course at the advanced undergraduate level on *Heat and thermodynamics* also carries three units of credit. The emphasis is on thermodynamics, which will include as careful a discussion of the laws and their many applications as is possible at this level. There can be a brief introduction to the third law. Perhaps the subject of thermodynamics is the best approach that physics can provide to critical thinking and analysis at the undergraduate level.

It is desirable to present at an early stage an experimental understanding of the absolute temperature scale; how the ice point, for example, is accurately established on the absolute scale with-

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[1] G. W. Carter, *The simple calculation of electrical transients* (Macmillan, 1944).

[2] H. F. Olson, *Dynamical analogies* (Van Nostrand, 1944).

out production of absolute zero temperature as a calibration point. This in turn requires a physical understanding of the concept of entropy, a difficulty that is commonly evaded in engineering and chemistry courses. Such an understanding requires a short exploration into kinetic theory of gases, going as far as a simplified derivation of the law of distribution of molecular velocities and the associated Boltzmann H-theorem, which so greatly illuminates the physical aspect of entropy. Mathematics beyond integral calculus is not required. Finally, it is desired to convey a picture of modern experimental thermodynamics, the production and measurement of temperatures near absolute zero, the approach to chemical equilibrium, and, if time permits, the analysis of molecular structure from specific-heat data.

The experimental facts of blackbody radiation provide a suitable introduction to the long history of attempts to account theoretically for its characteristics. Then the contribution of Planck to this field introduces the revolutionary concept of the quantum.

An elementary development of the principal features of the kinetic theory of gases will be a part of the course. Emphasis will be laid on the principle of equipartition of energy, including its application to visible particles as an illustration of its wide scope.

Physics of Solids

An advanced undergraduate course on *The physics of solids* was introduced partly because of the advances that are being made in the understanding of the solid state. However, the topics in this course are naturally more elementary than those usually associated with the theory of the solid state and perhaps could better be described by the title, *The properties of matter, particularly solid matter*. Among the topics that might be considered for this course are: elasticity, high pressure phenomena, plasticity, physical properties of crystals, superconductivity and low temperatures, physical principles in metallurgy, hardening and annealing, structure-sensitive and -insensitive properties, geophysics and ferromagnetism.

Advanced Laboratory

Our majors in physics will have had, beyond the laboratory associated with the particular beginning course they have taken, an intermediate laboratory course in which they carry out a number of selected experiments designed to give them familiarity with a variety of procedures. In the new advanced laboratory we propose to familiarize the students with a variety of technics. There will be instruction and practice in the use of machine tools and other shop equipment, so as to familiarize them with the principal tools and measuring devices, machinist's terminology, limitations of various working materials and so on. There will also be instruction in glass blowing, not primarily to give them the skill needed to make all their own research glass apparatus but to give them a feeling for what can and what cannot be done with glass. For example, it will avoid a situation in which an inexperienced graduate student asks the glass blower to construct a device in glass accurate to plus or minus 0.01 in.

An experiment will be set up in which the student will compare the indications of different types of vacuum gage such as the McLeod, the Pirani, the Knudsen and the ionization types. He will measure the ultimate vacuum obtained with a slow mercury diffusion pump and a fast oil diffusion pump and also investigate their speeds. Such an experience will be very valuable later on, should the man take on a research problem calling for vacuum technics. Part of this course may very well be devoted to a small number of carefully selected standard experiments in electronics to give a familiarity with the devices used and what can be done with them. This could well be followed by assigning to the student one or more problems calling for the design and setting up of an electronic device. To the extent that no detailed directions would be provided, it would take on the aspect of a small research. Part of the course could be used to familiarize the student with the use of the oscillograph and the main characteristics of amplifiers and scaling circuits. The construction of an electronic device for use in the research activities of the laboratory would be a justifiable activity provided that it did not take up too many laboratory periods.

It is not intended that the course shall consist entirely or almost entirely of technics. It is desirable that opportunity shall be given to carry through certain carefully selected experiments.

For instance, I believe that the measurement of the wavelengths of certain unknown spectrum lines by interpolation from the wavelengths of known lines impressed on the same plate would provide excellent practice in the use of a precision comparator, and would also show the need for using a calculating machine in place of a good slide rule. Such an experiment would perhaps be the most precise project the student had ever carried out, and he would learn that the accuracy of his work would be measured by the accuracy with which he could repeat his measurements. I would then follow this by an experiment in which the activity of a weak sample of polonium is measured by the ionization produced in successive equal intervals of time. The student would no doubt be surprised by the fact that his results were definitely not reproducible. This could be his introduction to the idea that certain measurements in nature are far from reproducible because of statistical fluctuations.

The next step would be to record the distribution in time of the pulses from a Geiger counter operated by a suitable beta-ray source; this would be a natural introduction to a Poisson distribution. An obvious extension of this, if time permits, would be to study the output of the same Geiger tube through a scaling circuit that yields fluctuations which no longer have the characteristics of a Poisson distribution. I believe that it is instructive to have students carry out, in succession, experiments such as these, in one of which it is possible to attain extraordinary precision, while in the other, lack of reproducibility is in no way caused by his carelessness in taking observations but arises from the nature of the observations themselves.

The details of this course have yet to be worked out. I believe that it has in it unusual possibilities as a method of instruction in experimental methods and may well turn out to be the most important change we have made. I have no doubt but that many graduate students who have not been our own undergraduates will find it advantageous to take this course.

In order to secure the best possible integration between these new courses, so as to avoid unnecessary repetition, it is considered desirable that each instructor be thoroughly familiar with the courses taught by his fellow instructors and possibly to plan his own set of topics with their advice or with the advice of some instructor whose responsibility it is to see that each course fits into the desired over-all program. This may not be workable, but it seems worth while at least to try it out. It is difficult to see how we can get a thoroughly integrated set of courses without more cooperation than usually exists. Frequently a course is organized and taught by an instructor without his colleagues knowing or caring what he attempts to cover. It may be that this free enterprise has advantages; it is certainly easy to carry out, but the question that remains is whether or not we can give the students a better deal by a reasonable amount of sane planning.

I know that our program is open to criticism because of the omission of courses on several topics such as mechanics, sound, electronics, conduction in gases, thermionics, photoelectricity and atomic structure. With regard to mechanics, for example, although the student does not get a systematic exposure to the subject, it is by no means completely ignored. For instance, the treatment of collision problems in the course on nuclear physics, of periodic motions in electricity, and of wave motion and other items in light, x-rays and electrons, can be regarded very definitely as at least keeping alive a student's contact with mechanics. Should any of our undergraduate students majoring in physics go on to do graduate work, they will at once get a one-year course in analytic mechanics.

If I am pressed to answer the question, Why do you not provide for courses in mechanics, sound, electronics? I will answer by asking another question. That question will be this: Do you agree with our premise that, under the boundary conditions within which we have to work, a student at the junior and senior level has a definite limit to what he can absorb and that it is not practicable to expose him during the last two years to more than, say, 250 contact hours of classroom instruction? If you do not agree, then it is not possible for me to answer. But if you do agree, then my answer will be that we can consider listing one or more of these courses, provided you can tell me just which course you propose to throw out to make room, and why the course to be admitted has a greater value than the one it displaces.

Secondary Shock Waves and an Unusual Photograph

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THE remarkable photograph reproduced in Fig. 1 was taken about 15 mi from Bitche, France, in the middle of March 1945, by M/Sgt Howard B. Gray, of the 208th Battalion, U. S. Field Artillery. The air was very clear, and the sky was of a deep blue with cirrus clouds that were estimated to be at an altitude of 20,000 to 30,000 ft. During the preparation firing prior to breaking through the Siegfried Line many men saw dark shadows in the form of huge arcs that passed across the white clouds, and were seen only where there was a background of cloud. Three of these arcs can be seen in Fig. 1. The centers of the arcs were on the German side of the Line, the direction of motion was away from the German positions, and the arcs ceased to be visible after they passed overhead. At first many men thought that the arcs were the visible sign of some new secret weapon, but the general opinion soon came to be that they were sound waves. This latter opinion is doubtless correct.

Another case of sound waves that were visible without any special equipment was observed more than 25 years ago by Lt. Thomas T. Mackie.[1] In that case the waves were passing through a heavy bank of fog that enveloped the artillery from which the waves came, and the observer was on a hill above the fog. The visibility of the sound waves was explained by Saunders[1] as follows.

Fig. 1. Sound waves seen against cloud.

> The muzzle wave from a large gun carries in its front a narrow region of compression immediately followed by a relatively wide region of expansion. . . . It would appear that the air was saturated with water vapor at a particular level, and that the expansion in the wave produced a visible increase in the fog density, the effect disappearing immediately again, owing to the subsequent re-evaporation when the air regained its normal pressure and temperature.

A different explanation seems more likely for the visibility of the wave fronts in Fig. 1. The clouds were so far away that the wave fronts of the light coming from them were practically plane, and the path of the light was deviated on passing through the compression at the front of the sound wave. That is, the visibility of the waves was probably produced in the same way as in Dvořák's[2] well-known and frequently employed modification of Töpler's "Schlieren" method.[3]

A striking feature in Fig. 1 is the multiplicity of arcs. The three that can be seen are about equally spaced, and the first and second are more distinct than the third. Why is the spacing so nearly equal? And unless they come from successive rounds of ammunition why is there more than a single wave front? The answer to these questions is suggested by considerations put forward in a recent article by Landau.[4] It is known[5] that it is only when changes of density are infinitesimal that a compressional wave can travel without undergoing an alteration of type. In fact, unless the waves are strictly plane a compression must be accompanied by a rarefaction, and the compression travels faster than the rarefaction. Now Landau points out that at some point in the region of rarefaction behind a shock wave there must be a minimum of pressure, and behind that minimum there is a region in which the compression is greater at greater distances to the rear. In this region the slope of pressure becomes

[1] F. A. Saunders, *Science* **52**, 442 (1920).

[2] V. Dvořák, *Wied. Ann.* **9**, 502 (1880).
[3] A. Töpler, *Pogg. Ann.* **127**, 556 (1866); **131**, 33 (1867).
[4] L. Landau, *J. Physics USSR* **9**, 496 (1945).
[5] See, for example, Lord Rayleigh, *Theory of sound* (Macmillan, ed. 2, 1896), vol. 2, pp. 33–36, 101.

steeper and steeper as time goes on, and soon a second shock wave is formed. "We arrive at the result that, at least at large distances . . . there exists not a single shock wave, as is usually assumed, but two shock waves following each other."

The conclusion that at a sufficient distance from a single shock there must be two shock waves, one following the other, has also been reached by DuMond and collaborators.[6]

The theoretical work of Landau and that of DuMond and his collaborators indicates that at a sufficient distance the abrupt rise of pressure in the second wave is just great enough to restore the pressure to that of the undisturbed medium, so that only two shock waves are to be expected. DuMond and his collaborators find that the pressure just ahead of the second shock wave is as much below that in the undisturbed medium as the pressure just behind the first shock wave is above it. If the rise of pressure in the first wave should exceed an atmosphere, it is clear that this situation could not exist, and DuMond and collaborators point out that their results are applicable only when the pressure elevation does not exceed a hundredth of an atmosphere.[7] Moreover, both Landau and DuMond treat the waves as nearly plane. If the approximations involved in both papers were removed, and if the shock were very intense, we may guess that the pressure reached in the second wave would be above atmospheric, and that after a shock wave has traveled a sufficient distance there might develop not only a second shock wave but also a third, and perhaps more. In Fig. 1 we may have a case where the first shock wave has given rise to a second, and that in turn to a third.

DuMond and collaborators apply their results to the waves around a shell or bullet that travels faster than sound. Photographs of such waves[8] usually show two shock waves, not only a bow wave but also a well-developed stern wave.

[6] J. W. M. DuMond, E. R. Cohen, W. K. H. Panofsky and E. Deeds, *J. Acoust. Soc. Am.* **18**, 97 (1946).

[7] In a study of shock waves in water, produced by the impact of small steel and aluminum spheres, J. Howard McMillen [*Physical Rev.* **68**, 198 (1945)] states that the pressure at the nose of a sphere reaches thousands of atmospheres.

[8] See, for example, D. C. Miller, *Sound waves, their shape and speed* (Macmillan, 1937), pp. 79–81.

Some 25 years ago Dayton C. Miller[9] obtained records of the sound from a 14-in. rifle. "The crack consists of a sharp, positive pulse followed quickly by a negative pulse of equal value and by a second positive pulse, the three pulses together lasting about 0.02 sec." Figure 48 in Miller's book is not very clear, but it looks as if the pressure in each wave rose to about one hundredth of an atmosphere. Each compression is followed by a definite rarefaction. This is a clear case of a pair of waves in which the pressure in the second rises to above atmospheric. Miller attributed the two compressional pulses to the bow and stern waves from the shell. But if one of the compressions arrives as much as 0.02 sec later than the other they must have been separated by a distance of something like 6.8 m. Unless the bow and stern waves fanned out a good deal before they reached the recording apparatus, it would seem that they could hardly be separated by any such distance.

The work of DuMond and his collaborators has a bearing on this question. They use the term "miss-distance" for the distance from the point of observation to the path of the projectile, and the term "wave-length" for the distance from the first to the second of the two shock fronts. They find experimentally that the bow and stern waves from a bullet do fan out, and their theoretical work shows that when the miss-distance is sufficiently great the wave-length is proportional to the fourth root of the miss-distance.[10] For a 0.50-caliber bullet their experimental work agrees well with this result. With a 40-mm bullet, which was the largest they used, the agreement was not as good.

It is possible to apply this fourth-root law to the case of the 14-in. shell investigated by Miller. Assuming that the shell was about 5 ft long, and taking other data from pages 118–119 in Miller's book,[8] we find that the fourth-root law gives a separation between the two waves of about 27 m. This is considerably more than the 6.8 m suggested above, but the difference may be because of the approximations employed in the theory, or because the waves have ceased to have the abrupt rise of pressure that characterizes shock waves.

[9] D. C. Miller, *Physical Rev.* **15**, 230 (1920); reference 8, p. 110.

[10] Landau's work also leads to this same relation.

The reason for the existence of a stern wave has not heretofore been clear. It has been suggested[11] that the stern wave is caused "by the closing together of the streams after they have passed the end of the bullet." It has also been suggested[12] that the stern wave may be a bow wave from the train of eddies behind the bullet. DuMond and his collaborators suggest that the stern wave "is initiated by the inrush of the air in the wake of the bullet to fill the void which its passage has created," and that "the disturbances from the nose and the tail of ordinary bullets *merely happen* [italics theirs] by a coincidence of the design ratios of length to diameter in ordinary bullets to cooperate in the formation of the head and tail discontinuities." Perhaps the explanation of the stern wave may really be that behind the bullet there is a region of greatly decreased pressure, and back of the pressure minimum we have the conditions for the forming of a shock wave.

Landau's work suggests an explanation for the three waves that appear in Fig. 1, and for the stern wave from a shell or bullet, and his work and that of DuMond and his collaborators may account for the time that elapsed between the two compressions that Miller observed at the passing of the "crack" from a shell.

[11] A. Mallock, *Proc. Roy. Soc.* (*London*) **A79**, 262 (1907), esp. p. 269.
[12] A. T. Jones, *Sound* (Van Nostrand, 1937), p. 28.

The Nuclear Physicists

These are the men who
Working secretly at night and against great odds
And in what peril they knew not of their own souls,
Invoked for man's sake the most ancient archetype of evil
And bade this go forth and save us at Hiroshima
And again at Nagasaki.

We had thought the magicians were all dead, but this was the blackest of magic.

There was even the accompaniment of fire and brimstone,
The shape of evil, towering leagues high into heaven
In terrible, malevolent beauty, and, beneath, the bare trees
Made utterly leafless in one instant, and the streets where no one
Moved, and some walls still standing
Eyeless, and as silent as before Time.

These are the men who
Now with aching voices
And with eyes that have seen too far into the world's fate,
Tell us what they have done and what we must do.
In words that conceal apocalypse they warn us
What compact with evil was signed in the name of all the living,
And how, if we demand that Evil keep his bargain,
We must keep ours, and yield our living spirits
Into the irrevocable service of destruction.

Now we, in our wilderness, must reject the last temptation:
The kingdoms of earth and all the power and the glory,
And bow before the Lord our God, and serve Him
Whose still small voice, after the wind, the earthquake,
The vision of fire, still speaks to those who listen
And will the world's good.

—From *Ultimatum for man*, by PEGGY POND CHURCH (Ranchos de Taos, New Mexico).

A New Impact Apparatus*

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FOR the demonstration of multiple impact and collision phenomena two types of apparatus are ordinarily used. The first is essentially a row of pendulums with bifilar suspensions. The second consists of a number of spheres rolling on a double track or in a groove. This paper presents a third type. Before describing it in detail, we will point out the advantages and disadvantages of the various types.

With the first two types the velocity of a sphere before collision may be predetermined by releasing it from a given level, and the velocity after impact is measured in terms of the final level to which the ball rises. Usually this is not actually done with the second type, but it may be achieved by inclining the end sections of the track; thus a ball is released on one incline, collision occurs on the level part of the track, and a ball then rolls up the other incline. With the new apparatus the velocity cannot be predetermined.

Only with the pendulum type is it possible to demonstrate interesting sequences of events, such as those discussed by Lemon[1] and by Chapman,[2] which students find fascinating and which are instructive if studied analytically. However, this type has the serious disadvantage that painstaking adjustment is necessary to establish and maintain accurate alinement of balls, and it is difficult to assure collinear impact. With the second type no time has to be spent on alinement; but the phenomena are confused by the presence of rotational as well as translational momentum, and serious errors are introduced by friction on the track and between adjacent balls; moreover, balls of mixed sizes cannot be used.

The main advantages of the new apparatus are that alinement is automatically assured, rotation and friction between balls are absent, and many experiments can be performed with it that are virtually impossible with conventional apparatus.

Apparatus

As illustrated by Fig. 1, eight pool balls, each with two parallel holes, are supported by two tightly stretched, heavy piano wires mounted in a hardwood box open on one side. The back of the box is painted white inside to form a bright background against which to observe the motions of the darker balls. The back is made of $\frac{3}{4}$-in. hardwood lumber so as to strengthen the box, which is under great tension from the wires. Actually only one wire is used; at one end it is welded to a bolt b_1 protruding through the end of the box, is then threaded through a hole h_1 in a flush plate p at the other end, then threaded back through hole h_2 and welded to bolt b_2. By means of wing nuts w on the bolts the tension in the wires can be varied. Inside the box the two bolts are connected by a peg which prevents rotation of the bolts and consequent twisting of the wires. There is a rubber bumper r at each end of the box.

Experience with an earlier model has shown that when the apparatus is not in use the wires should be loosened; otherwise the box will be badly warped because of the great tension. Furthermore, it seems best to take the weight of the balls off the wires during storage by setting the box on end; hence the rubber feet indicated at f.

There are three main reasons why two wires are better than one: when there is only one wire the points of contact during collision are along the edges of the holes, and under these conditions the balls seem to crack rather easily; balls are sometimes set to spinning around a single wire, and obviously this is undesirable; there is less sagging due to the weight of the balls.

The triangles t represent wooden blocks whose base equals the diameter of a ball. They are

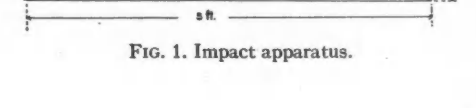

FIG. 1. Impact apparatus.

* Part of this paper is summarized in an abstract, *Am. J. Physics* 11, 47 (1943).

[1] H. B. Lemon, *Am. J. Physics* (*Am. Physics T.*) 3, 36 (1935).

[2] S. Chapman, *Am. J. Physics* 9, 357 (1941).

used as markers to indicate the position of the balls before impact.

One ball is indicated at *m* as having a block clamped on it which serves as an auxiliary mass. Two such blocks are used having masses equal, respectively, to one and two times that of one ball. As Fig. 2 indicates, each block is dissectible and consists actually of two half-blocks clamped together on a ball by means of bolts and wing nuts. It can therefore be mounted on a ball or removed easily and quickly.

Some of the balls have holes of small diameter drilled through their centers in a direction perpendicular to plane of the wires. By threading fishline through these holes and then connecting the ends, two or three balls may be fastened together to produce bodies having double or triple masses.

Pool balls[3] appear to have a coefficient of restitution of approximately 0.7, as measured by standard methods. The coefficient of sliding friction is about 0.16. Pool balls were chosen, rather than, say, steel balls—which have more desirable elastic constants—because their weight is relatively small. It was desired to make the apparatus large enough so it could easily be seen from the rear of a large lecture room; this would have been extremely difficult with heavy metal balls.

We have also tried using cylinders, both steel and hardwood, instead of spheres. They yielded completely anomalous results[4] and were therefore not usable.

TABLE I. Suggested sequence of experiments with spheres of equal mass.

	Series 1		Series 2		Series 3		Series 4		
	M	*S*	*M*	*S*	*M* →	← *M*	*M* →	*S*	← *M*
(*a*)	1	1	1	1, 2, 3	1	1	1	1	1
(*b*)	2	2	2	1, 2, 3	1	2	1	1	2
(*c*)	3	3	1	1, 2, 1, 2	1	3	1	1	3
(*d*)	1	2	2	1, 2, 1, 2	2	3	1	2	3
(*e*)	1	3	3	1, 3	2	6	1	1, 1	3
(*f*)	2	6	4	1, 3	1 ⟶	⟵ 1	1	1, 2, 1	3
(*g*)	5	3	5	1, 2	1 ⟶	⟵ 2	2	3, 2	1
(*h*)	5	1	6	1, 1	1 →	⟵ 2			
(*i*)	7	1							

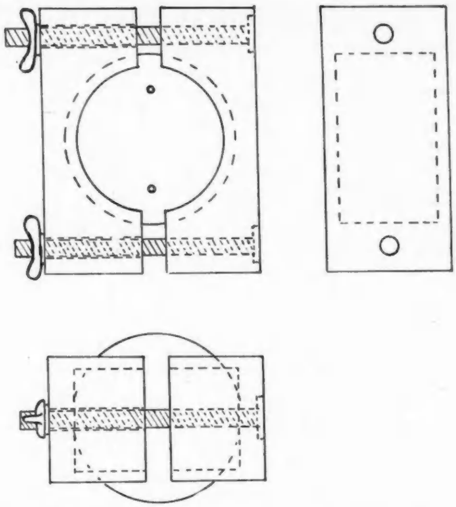

FIG. 2. Dissectible block to increase effective mass of ball.

Experiments

Spheres of equal mass.—A sequence of experiments is suggested in Table I. The experiments fall naturally into four series. Series 1 comprises the usual demonstrations carried out with conventional apparatus. In the table the letters *M* and *S* refer, respectively, to "moving" and "stationary." Thus series 1, line (*a*) refers to the experiment in which 1 ball moves toward 1 stationary one; 1(*b*) means 2 balls moving toward 2 stationary ones; 1(*c*) means 3 moving balls and 3 at rest. Balls are propelled simply by hand.

These demonstrations show that, to a first approximation, when any group of moving balls having equal masses collides with a like group that is stationary, the latter moves on while the former comes to rest. Most students are perfectly satisfied and content with this observation. Some, however, are not. They notice that, when position markers are used, it is apparent that both groups move after impact, though evidently one very much more slowly than the other. One is brought to rest by friction within a fraction of an inch, while the other moves much farther. Of course, the fact that neither group actually has zero velocity after collision may be attributed

[3] New pool balls are rather expensive. However, commercial pool rooms often have on hand a supply of discarded balls which have become slightly chipped or otherwise damaged; they are quite satisfactory for present purposes and can be bought cheaply.

[4] These "anomalies" have been under investigation experimentally in our laboratory and theoretically by our colleague, Professor John J. Gibbons. However, their discussion now would not be consistent with the purposes of this paper.

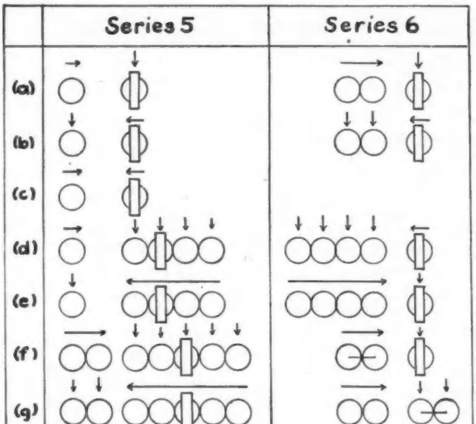

FIG. 3. Schematic representation of experiments with bodies of unequal mass.

to imperfect elasticity; however, it is not at all evident that this is the only possible explanation.

Experiments 1(*d*) to 1(*i*) involve groups consisting of unequal numbers of balls. They show, again to a first approximation,[5] that no matter how many spheres are at rest or moving before collision, the number moving afterward is the same as before. For experiments 1(*d*) to 1(*f*) the number of moving bodies is smaller, and for 1(*g*) to 1(*i*) it is larger than the number of those at rest before collision.

The demonstrations comprising series 2 cannot be performed satisfactorily with the usual apparatus. Here the stationary balls are distributed along the wire in discrete groups. Thus in 2(*a*) there is one ball at rest, a few inches from it a group of two at rest and in contact with each other, and still farther on a group of three at rest and in contact. A single ball is projected along the wire toward these three groups, and, as is to be expected from earlier experiments, after all collisions one ball is moving, while groups of one, two and three are still at rest. Clearly, one unit of momentum passes through the whole array. Other demonstrations in this list are merely variations of this first one, making use of different groupings. Thus 2(*c*) and 2(*d*) involve four groups at rest. In 2(*e*) to 2(*h*) the number of moving balls exceeds the number in any stationary group.

In the experiments of series 3 no bodies are at rest; instead, groups are propelled toward each other. For instance, in 3(*e*) two balls move toward the right and six toward the left; after the collision, two balls still move toward the right and six toward the left—though, of course, not the same ones as before impact.

The arrows appearing in 3(*f*) to 3(*h*) indicate velocities of different magnitude before impact. It is rather important to demonstrate, as in 3(*g*), that if before impact one ball is moving with a large positive velocity, and two with a small negative velocity, after the impact there will still be one ball with a large positive velocity, and two with a small negative velocity.

The experiments in series 4 combine the features of those in series 2 and 3. To illustrate, for 4(*g*) there are two separate, stationary groups of three and two balls, respectively. Toward these are projected two balls from the left and one from the right. After the collisions there will be two stationary groups—as before—as well as a pair of balls moving toward the right and one toward the left—as before.

Experience has shown that this whole sequence of 32 demonstrations, which lead from simple to complex situations, and illustrate many possible combinations, is unusually helpful in making clear the basic principle of conservation of momentum. The student sees clearly that, at least in cases involving perfect or near-perfect elasticity, the momentum is the same after collisions as it was before, no matter how many bodies may be involved or how many collisions may occur. One advantage of this apparatus is that it is always in adjustment and requires no waiting for the balls to come to rest; the whole sequence can easily be shown in ten minutes.

Bodies having unequal masses.—The demonstrations suggested thus far show that certain interesting phenomena occur and that momentum is conserved *when the balls have equal masses.*

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[5] This means, as in the preceding paragraph, that no single ball or group of balls is actually at rest immediately after collision. However, some come to rest so soon that without the use of position markers their motion might not be noticed at all. This proviso applies to practically all subsequent experiments, though to avoid tediousness of exposition it is not repeated in the text hereafter.

An interesting question arises here in regard to effective teaching. In an elementary physics class should attention be called, deliberately, to such approximations? The superior student would find it stimulating, the poor one confusing. Hence . . . .?

They do not show what would happen, or what would *not* happen, if the masses were unequal.

To study the effect of inequality of masses a block is clamped on one ball to provide a body having twice the mass of a ball. It is suggested that a ball near the middle of the array be chosen. Demonstrations of basic interest are indicated schematically by Fig. 3. Vertical arrows indicate bodies stationary before impact; horizontal arrows, bodies moving before impact.

Unfortunately, it is not possible with this apparatus to confirm quantitatively the theoretical predictions for collisions between single objects of unequal mass. It is possible, however, to show that the phenomena observed are qualitatively consistent with theory. Thus for experiment 5(a) theory predicts, assuming elastic collision, that $v_1=-\frac{1}{3}u_1$ and $v_2=+\frac{2}{3}u_1$, where v_1 and v_2 refer to postcollision velocities of the single and double masses, respectively, u_1 and u_2 to their antecollision velocities, and the negative sign to a direction opposite to that of u_1. As predicted, the two balls move in opposite directions after impact, the ball of larger mass having the larger velocity. However, the velocities clearly are not in the ratio of 1 to 2. This may be explained, at least in part, by assuming a coefficient of restitution of 0.7, for which theory predicts values of $v_1=(4/30)u_1$, and $v_2=(17/30)u_1$.

Qualitatively, the important point for the student to recognize is that when the balls are unequal in mass, as in experiments 5(a) and 5(b), one ball does not upon impact set the other into motion while coming to rest itself, as was the case in experiment 1(a). Nor do they simply exchange velocities in 5(c), as they did in 3(a). Nor in 5(d) does one ball move off to the right after collision, while all the rest remain at rest. The presence of the double mass in the line "spoils things."

The results just noted usually do not surprise the student. However, trouble appears in experiment 5(f). Here one might well expect that the two moving balls a and b should pass on their momentum to balls c and d, that these would in turn transfer theirs to the double mass which could then pass on the double momentum to the last pair, f and g; therefore, that f and g should move on while all the others remain behind at rest. Likewise, in experiment 5(g) it would seem that after collision two balls should remain stationary, while all others move on. However, things do not happen that way at all, and it becomes necessary to demonstrate that a pair of balls of equal masses is not equivalent to one double mass, so far as impact phenomena are concerned. Experiments 6(a) and 6(b) prove this point conclusively, while 6(d) and 6(e) yield additional evidence.

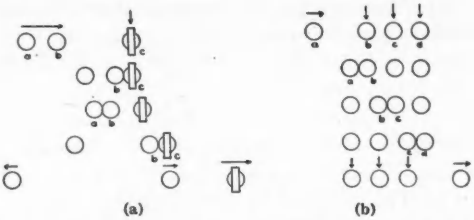

FIG. 4. Illustrations of the use of the "method of sequence of events."

In experiment 6(f) balls a and b are tied together as described in the section on apparatus. When this connected pair of spheres is projected toward the double mass c, the former comes to rest during impact while the latter is given an impulse forward. Apparently, then, two connected balls may be considered as a double mass, though unconnected ones may not, even though the latter may seemingly be in contact during motion. Finally, in experiment 6(g) the connected pair is substituted for the double mass, as in experiment 6(a). The same results appear.

Similar experiments may, of course, be made with triple masses produced either by tieing three spheres together, or by clamping a block of double mass on one ball.

Discussion

Too often students are left with rather hazy ideas as to the logic of these experiments. It would seem that in the interest of intellectual honesty, if for no other reason, the following facts should be stated unequivocally.

(1) It is correct to say that when two perfectly elastic bodies collide, the velocities after impact may be predicted by invoking simply the principles of conservation of momentum and energy.[6]

[6] For a comprehensive treatment of two-particle collisions, see R. H. Bacon, *Am. J. Physics* **8**, 154 (1940). A

(2) When groups of bodies are involved in a collision and the members of a group are *not in contact*, it is possible to predict effects "on the basis of sequence of events," as Chapman's beautiful experiments[2] have shown convincingly. This simply means that a multiple collision may be considered as a succession of two-body impacts. The method is illustrated in Fig. 4(a). The collision of two separated balls a and b with the ball of double mass c is regarded as consisting of three separate impact events with different momentum distributions before and after each event. The first event is the impact of ball b with ball c. After this event a still has its original velocity u, b a velocity equal to $-\frac{1}{3}u$, and c a velocity equal to $+\frac{2}{3}u$, as computed in the standard way. The second event is the impact of a and b, after which $v_a = -\frac{1}{3}u$, $v_b = u$ and $v_c = \frac{2}{3}u$. The third event occurs when ball b catches up with and strikes the slower ball c; the velocities then become $v_a = -\frac{1}{3}u$, $v_b = \frac{5}{9}u$ and $v_c = \frac{8}{9}u$.

(3) When a group of bodies in contact is involved in a collision, the postimpact motions of individual members of the group cannot be predicted by appealing simply to the principles of conservation of momentum and energy.

It might seem at first thought that this conclusion does not apply to experiments 1(b) and 1(c), in which the colliding groups have equal numbers of spheres. Often in such cases pairs and triplets of balls are treated as equivalent to single bodies having double and triple masses, respectively. While this procedure appears to yield correct predictions, it is bad logically because other experiments, such as 6(a), seem to show that a pair of spheres that are not rigidly connected is not equivalent to a single body with double mass.

It is well known, though apparently not too widely recognized, that for, say, a particular three-body collision the set of equations expressing the principles of conservation of momentum and energy has an infinite number of solutions. While this means that in any particular case these principles do not demand a unique postcollision momentum and energy distribution, it also means that within limits no distribution is ruled out. Therefore, our conclusion, derived from experiments, that two unconnected bodies in contact cannot be considered as equivalent to a single body with double mass, is not a logical consequence of the conservation principles.

(4) The method of sequence of events discussed above is useful in predicting, at least qualitatively, the results obtained with our apparatus in all the experiments described in this paper, even when groups of spheres are in contact[7]—though we know of no adequate theoretical reason why this should be true. Thus, first-order momentum effects of experiment 6(a), in which two small masses are in contact before impact, are predicted by treating the pair as if they were not touching and proceeding as in Fig. 4(a). Likewise for experiment 1(e), the triplet of balls, which certainly appear to be touching each other, are simply treated as separated balls and the collision as a sequence of separate and independent events, as illustrated by Fig. 4(b).

The use of this method involves the assumption that *to a first approximation a group of spheres which touch each other acts during collision as if each sphere were independent of the others, as if they were not in contact but were separated by small distances, thus permitting a succession of single impacts to take place one after another in time.* This is a working assumption independent of and in addition to the principles of conservation of momentum and energy.

Of course, the method of "sequence of events" does not, in the dynamical sense, constitute an explanation of what happens, or why, when balls are in contact. To clarify this matter further research is required, both experimental and theoretical.

careful analysis of basic concepts and principles, suitable for beginners, is given by L. W. Taylor, *Physics—the pioneer science* (Houghton Mifflin), pp. 200–216.

[7] This is illustrated convincingly by the experiments described by J. Satterly, *Am. J. Physics* **13**, 170 (1945).

Think like a man of action and act like a man of thought.—Henri Bergson.

The Treatment of Extended Light Sources in Elementary Textbooks

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AND

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THE point source has long held an unwarranted monopoly of attention in textbooks that discuss elementary optics; extended sources are seldom mentioned. Such a treatment leaves the student totally unprepared for most practical problems, in which sources are usually extended. The only example of a point source in common use appears to be the photometer bench, and its possibilities for discussion are soon exhausted. Then, since further treatment of the point source is redundant and the student is unprepared to consider extended sources, the text proceeds to other topics. The student is thus left with the impression that the only use of a light source is comparison against another light source.

The value of the concept of the point source to physics is not that it occurs frequently in practice, but that it is the first step in computing the illumination provided by extended sources. The author who fails to indicate that the point source is a means and not an end is misleading his readers, and is supplying evidence to support the notion that physics deals only with impractically idealized problems.

Once the extended source has been treated, there is a wealth of exemplifying material available "to lend artistic verisimilitude to an otherwise bald and unconvincing narrative." Natural sources—the sky, windows, and so forth—are indubitably extended, and artificial sources are tending toward extended areas of low brightness. In addition, two very practical, interesting, and instructive examples may now be treated, for the film in a camera and the retina of the eye are each illuminated by a lens which must be regarded as an extended source.

The advantages of the treatment of extended sources in elementary textbooks are not limited to the pedagogic convenience and esthetic satisfaction of the author, or to vivification of subject matter and inculcation of general scientific culture for the student. The concepts developed are practically useful and easily retained for everyday use. The formulas, of course, are commonplace in more advanced textbooks, but they are so simple and useful that they deserve to be among the tools available in the daily life of the nonspecialist.

The form of the treatment of the extended source will vary, depending upon whether or not the author permits himself the use of the calculus. If he does, the development followed in most advanced textbooks may be adopted.[1] If the author has not been using the calculus, there is still a cogent argument for introducing the concept of integration at this point; namely, that the calculation of illumination from an extended source is a most excellent opportunity to introduce the student to the need for the methods of the calculus and the physical significance of integration procedures. The correspondence between the physical interpretation and the mathematical step is so close that the student who has gained a clear mental picture of the physical procedure is well on his way to understanding the mathematical method.

For those authors who prefer to avoid the direct use of the calculus, the derivation below leads to the most important formula by methods no more nefarious than the student has already encountered in secondary school algebra and geometry. It is fortunate that the most important case of illumination from an extended source—the circular disk—is also the easiest to calculate. This case is most important because it applies to most optical instruments. For example, the film in a camera is illuminated by a circular disk, the lens; and the lens may be shown[1] to have the same brightness when seen from a point on the film as the object being imaged at that point.

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[1] Hardy and Perrin, *Principles of optics* (McGraw-Hill, 1932), p. 410.

In Fig. 1, suppose it is desired to compute the illumination at the point A on the plane ACF due to the disk shown. The disk has a uniform brightness B (ca/cm^2) when viewed from any direction; that is, it obeys Lambert's law. The plane ACF is parallel to the xy-plane, which contains the disk with center at the origin O. The point A lies on the z axis.

To regard the disk as a point and apply the inverse-square law directly would be too gross an approximation because: (i) the outer parts of the disk are farther from point A than the central part; (ii) light from the outer part of the disk which arrives at A leaves the disk obliquely and arrives at the plane ACF obliquely. These two facts suggest an approach to the problem.

If the whole disk were divided into small elementary areas s, then each element could be considered a point source of intensity Bs in the direction of the perpendicular sP, and of intensity $Bs\cos\theta_3$ in the direction sA. This ray arrives at A at the angle θ_3 also, and the illumination at A due to s is therefore $Bs\cos^2\theta_3/d_3^2$. The total illumination at A will be the sum of the light from all the elements into which the disk was divided.

The summation process can be shortened by noting that all the elements in an annular ring such as *stuv* have the same values of d and $\cos\theta$. Thus the illumination at A due to the third ring will be $B\cdot 2\pi(r_4^2-r_3^2)\cos^2\theta_3/d_3^2$, where the area of the elementary ring $2\pi(r_4^2-r_3^2)$ has replaced the area s of the small element. The areas of the rings can all be made the same $[=\pi r_1^2]$ by choosing $r_2=\sqrt{2}r_1$, $r_3=\sqrt{3}r_1$, $\cdots$. If the entire disk is divided into n annular rings, then $r_1=R/\sqrt{n}$. The total illumination E at A will then be the sum of the contributions from the various rings:

$$E=\frac{B\cdot\pi r_1^2}{d_0^2}+\frac{B\cdot\pi r_1^2}{d_1^2}(\cos^2\theta_1)+\frac{B\cdot\pi r_1^2}{d_2^2}(\cos^2\theta_2)+\cdots. \quad (1)$$

This expression can be simplified by noting from Fig. 1 that $d_1^2=d_0^2+r_1^2$, $d_2^2=d_0^2+2r_1^2$, and so on. Also it will be seen that $\cos^2\theta_1=d_0^2/(d_0^2+r_1^2)$, $\cos^2\theta_2=d_0^2/(d_0^2+2r_1^2)$, $\cdots$. Then, taking out-

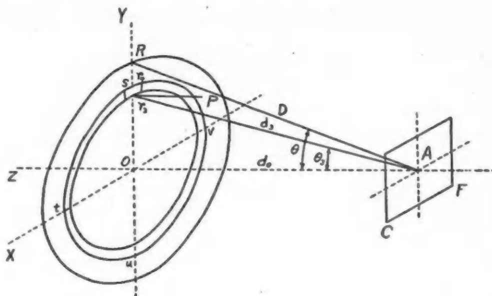

FIG. 1. Illumination due to a disk source.

side the brackets the constant factor $B\cdot\pi r_1^2=B\cdot\pi R^2/n$, there results:

$$E=\frac{B\cdot\pi R^2}{n}\left[\left(\frac{d_0}{d_0^2}\right)^2+\left(\frac{d_0}{d_0^2+r_1^2}\right)^2+\left(\frac{d_0}{d_0^2+2r_1^2}\right)^2+\cdots\right]. \quad (2)$$

When n is a large number (that is, when the disk has been divided into many narrow rings), the terms inside the brackets add up to $n/(d_0^2+R^2)$, as may be proved by more advanced methods. There results:

$$E=B\cdot\pi\left(\frac{R^2}{d_0^2+R^2}\right)=B\cdot\pi\sin^2\theta, \quad (3)$$

where E is in lumens per unit area when B is in candles per unit area.

The middle member of Eq. (3) is a generalization of the usual inverse-square law, in which the perpendicular distance d_0 has been replaced by the distance D measured to the edge of the disk. However, it is generally more useful to regard the illumination as explicitly independent of distance and dependent only upon the angle θ, as shown in the right-hand member of Eq. (3). Thus the illumination at a point is determined completely by the cone of light that converges upon it. The solid angle subtended by the disk at A is given by $\omega=4\pi\sin^2\frac{1}{2}\theta$, and for values of θ up to 25°, $\omega=\pi\sin^2\theta$, with an error within 4 percent. Thus for many practical cases the illumination

at a point is determined by the solid angle subtended by the source.[2]

This solid-angle approximation is useful because it allows the calculation of illumination by noncircular sources. Even when $\theta=\frac{1}{2}\pi$, this approximation is in error only by a factor two; and, in this special case of a luminous plane extending to infinity in all directions, the rigorous form $E=\pi B$ would be used. Such a situation occurs practically for points very close to large, perfectly diffusing glass windows, or for points on the level ground illuminated only by an unobstructed overcast sky of uniform brightness.

The error of the solid-angle approximation is often unimportant in illumination calculations since the two most common light receptors, the eye and the photographic emulsion, have a logarithmic response. Thus, even if the error should approach the limiting value of a factor two, the exposure of the photographic film would be known to the nearest stop setting. This is sufficiently precise for most photographic purposes.

The camera affords an excellent practical example of the use of Eq. (3). The illumination at an axial point on the film is the result of a circular cone of light from the lens. The size of this cone is controlled by the iris diaphragm. The illumination on the film depends also on the distance of the film from the iris diaphragm (strictly, the exit pupil), since θ is a function of that distance. It is therefore clear that no system of aperture markings (such as the $f/$-number) which fails to take account of the object distance can be an accurate measure of the light-gathering ability of the lens under all conditions. Thus Eq. (3) emphasizes the insufficiently known fact that the $f/$-number is defined for infinitely distant objects, and needs to be corrected when used for nearby objects.

Another example of the use of Eq. (3) is for the consideration of the illumination on the retina of the eye. The cone of light converging upon a point on the retina is limited by the pupil of the eye. Since this cone cannot be increased in size by any external arrangement, it is clear that no optical instrument can increase the illumination of the retina or, thereby, the apparent brightness of any extended source. On the other hand, an external instrument can decrease the apparent brightness of an extended source if the instrument restricts the cone to a size less than that fixed by the pupil of the eye.

These considerations do not apply when viewing a point source, and the apparent brightness of a point source can be increased by a lens system. Thus stars may be seen in daylight through a telescope since the apparent brightness of the star is increased by the lens while the apparent brightness of the extended sky is not increased.

The authors wish to express their thanks to Professor A. C. Hardy for many helpful discussions.

[2] This fact was used as a basis for a system of lens aperture markings in F. M. Steadman, *Unit photography* (Van Nostrand, 1914).

I VALUE in a scientific mind, most of all, that love of truth, that care in its pursuit, and that humility of mind which makes the possibility of error always present more than any other quality. This is the mind which has built up modern science to its present perfection, which has laid one stone upon the other with such care that it today offers to the world the most complete monument to human reason. This is the mind which is destined to govern the world in the future, and to solve the problems pertaining to politics and humanity as well as to inanimate nature. It is the only mind which appreciates the imperfections of the human reason, and is thus careful to guard against them. It is the only mind that values truth as it should be valued and ignores all personal feeling in its pursuit.—H. A. ROWLAND.

Physics in General Education: The Challenge to the Physics Teacher

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I WISH to discuss two of the "requisites" mentioned by Dean Cooper,[1] namely, the last clause of requisite 3 and requisite 2. In the first area, ". . . influence [of basic scientific principles] on development of thought and institutions," physics has played a profound and determinative part, though only recently have physicists begun to be aware of the fact. In the second area, "physics and responsible citizenship," physicists, along with their colleagues in the other sciences, have, at least until recent months, clearly and catastrophically failed to measure up to their responsibilities. I shall take up these two points in order.

The paradoxical position of our science in modern culture, and especially in the liberal arts curriculum, is highlighted by the fact that even today the general public does not quite know whether to associate physics with bottles, books or atom bombs. For this condition we have only ourselves to thank. As press agents for our professional field we have been—good physicists. We typically, and I fear all but unanimously, think of physics as not quite belonging in the liberal arts curriculum. Certainly we fail to impress on our students the overwhelming extent to which physics has "influenced the development of thought and institutions." We could scarcely fail to do this if we were ourselves acutely aware of it.

Instead, we typically, though no doubt somewhat unthinkingly, consider physics as something resembling a pre-engineering subject or—equally narrowly—as primarily a discipline for budding physicists. Our colleagues in the humanities, all too eager to take us at our own evaluation, are today filling their journals with alarmist articles on the general theme that increased interest in the sciences, stimulated by scientific contributions to the war, will act to the disadvantage of the humanities. College administrators, who for some reason seem to be drawn largely from the humanities, think of physics as a subject somehow outside of the cultural area which the liberal arts serve. We physicists, having made a bed of this kind for ourselves, must now perforce lie in it.

The tragic part of this outgrowth of our own intellectual provincialism is that it utterly belies the facts underlying the whole history of science. It is the least of the disadvantages of this provincialism that it is placing a handicap on the development of physics. The real tragedy is that it is placing a handicap on the development of the liberal arts. Even the broadest among us are inclined to think that physics began about the time of Galileo and was really given its character by Isaac Newton and a few contemporaries. In taking that position we play directly into the hands of those who associate physics with technology and who therefore look to earlier times for the real roots of our culture, in their own minds excluding physics as one of the possible sources of this culture.

Now it is true that the real roots of our culture do penetrate far beyond the sixteenth century. But contrary to the popular impression, even among physics teachers, in that early culture medium is to be found the real germ of physics, along with those of the other major sciences as well. Charles Singer describes it this way:[2]

> Far back in the history of Greek thought we see men feeling their way to an interpretation of that universal principle which they distinguish as *physis*, a word which survives in our modern terms *physics*, *physiology*, *physical*, *physician*, etc. *Physis* meant at first *growth* or *development*, the essential element of all existence, and it was specially applied to living things. Gradually there dawned the idea that this growth followed definite rules which differed in different cases, but in which a common character might be distinguished. By a simple process of transference, *physis* came to be regarded as this rule or manner of development itself, and so came to mean something near to what we now call a *natural law*.

It is scarcely necessary to particularize on the Greek contributions to the early development of

[1] R. M. Cooper, "The requisites of general education," *Am. J. Physics* **14**, 387 (1946).

[2] C. Singer, *Religion and science* (Benn, London, 1928), p. 11.

science. They had their beginning in the Greek rescue of the quest for knowledge from the position of handmaiden to theology. Several men stamped into the body of early science the characteristics that were later to distinguish it sharply from other branches of the intellectual enterprise. One of the most profound of these was Thales of Miletus. He plumbed the depths of Egyptian technology and turned it into the broader intellectual pursuit that ultimately became science. Pythagoras was the first to recognize the necessity for leaving permanent records of observation of natural phenomena and of establishing some clearing house for them. Heraclitus developed the concept of universal law, with its basic connection between cause and effect which is the keystone of the scientific structure. Democritus was the father of atomism; and who will maintain that, with all its crudities, atomism anticipated less of the future than does our current scientific doctrine, at least prior to the era of atomic power?

At the same time, in an adjacent territory, was occurring a parallel and at least equally important concept. The Jewish-Christian body of doctrine, while it was handicapped by being more of the nature of the handmaiden of theology, nevertheless was the pioneer in the concept of a God of law who, with certain infrequent lapses termed miracles, obeyed his own law, which the Greek gods did not. In the union of the Greek with the Jewish-Christian intellectual tradition, science as we know it was conceived. But conception is only a prelude to actual birth. In this case the period of gestation lasted for more than a thousand years. Not until the sixteenth century did parturition actually occur. And, once born, the most prominent trait of the new infant was found to be his predilection for physics and astronomy.

The subsequent developments were epochal. Their significance lay not so much in the *effects* of science, much as we like to stress them, as in the *nature* of science. It is utterly unique. Literature and the arts have been produced principally by special geniuses, and the rate of such production appears neither to grow nor to improve with passing time. Current masterpieces of art and literature are of no greater merit, nor are they being produced any more profusely today in proportion to the population, than the corresponding products of 2000 years ago. In the sciences, on the other hand, is to be found the first large body of knowledge that is both sequential and cumulative. As a unified army, organized for a sustained assault upon the citadel of human ignorance, there has been nothing to compare with the sciences in the whole recorded development of human thought. It is possible to question the value of the material that the sciences discover; much of it seems trivial to the lay mind. One may also be fearful of the ultimate effect of scientific philosophy on human welfare; many thoughtful men hold science responsible for some of the major ills of the day. But whether for good or for evil, the fact that science dominates modern thought cannot be disregarded.

Teachers of physics are not doing as much as they can to cultivate this rich heritage, for it is in physics that the heart of the development of science is to be found. This omission is the more regrettable in that relatively little additional time is required, very little in proportion to the values derived. It is rather a matter of the mode of presentation of the subject than of added time for unrelated material. The great bulk of the material is already before our classes in the usual course of conventional physics instruction. It is indeed more completely in hand than it can ever be in courses in the history or philosophy of science. All that is required to bring out the significance of the discoveries being described is the deft manipulation of relative emphases which any skillful teacher uses, consciously or unconsciously, as part of his stock in trade. No physicist who has a vision of the prominent part played by his subject in the general intellectual enterprise need make any apology to the humanities for the breadth that physics can impart to education. But the number who have that vision is disconcertingly small, and still smaller is the number of those who are willing to go out of their way to acquire it.

The fact is that physics has lent its character to all disciplined thought. It is trite to say that we live in the scientific era. It is not trite, and it would be even more accurate to say that we live in an era whose intellectual structure has been shaped by physics. This is not so much because of the multiplicity of inventions that make

the world so different and life so much safer and easier than it was a century ago. It is rather because of the subtle conception which gave these gadgets birth and which is vastly encouraged by their use: *man's confidence in his intellectual supremacy over nature.* And who will say that this is not a proper theme, indeed that it should not be one of the *central* themes, in any liberal arts curriculum?

Physics is pre-eminently qualified, above all the other sciences, to illustrate this fundamental aspect of the general intellectual enterprise. Physics has a clear and overwhelming priority among those disciplines that shaped the prevailing modes of thought in this, the scientific era. In company with astronomy which, both in subject matter and in technic, has always been a specialized department of physics, it met and withstood the full impact of authoritarianism. It thus earned the double distinction of being the pioneer in the most productive intellectual field of all time and of being the successful champion of the scientific idea against the tremendous odds in favor of those who denied its validity and who invoked every agency at their command to destroy it.

Yet what are we doing as teachers of physics to cultivate this rich heritage? Almost literally nothing. There is, to be sure, little lack of lip service to these broader concepts of the profession of teaching physics. It is when we examine the textbooks that are used, the actual proportion of the students' study time that is assigned to the pursuit of these aspects of science, and the amount of attention given to them in examinations that one realizes in what low regard they are actually held by the typical physics teacher. For, make no mistake, the importance that will be attributed to these elements of science by students, and indeed the measure of the teacher's own confidence in them as elements of physics instruction in liberal arts, is the proportion of emphasis that he gives to them through the conventional channels of instruction. He is in fact only self-deceived when he says, "Oh yes! I regard these as very important and I mention them in my lectures," and then leaves them almost completely out of the actual corpus of his instructional program.

The prevailing attitude of almost complete indifference on the part of physics teachers to the broader aspects of their science is perhaps partly understandable. The frontier of discovery is aflame with new and exacting advances. Physicists, and equally all men of science in general, consider themselves too busily engaged on the scientific battle front to give serious attention to explaining to visitors the larger aspects and significances of the campaign, even in the case of those unusual men who themselves know what these are. And as for the basic issues and history of the scientific war itself, that seems to most science teachers a hopeless undertaking, even if they have given thought to those issues, which most have not. Yet it is exactly these elements which can make the study of physics of maximum value to the majority of general students, and which the liberal arts curriculum has correspondingly a right to demand.

But if, through some miracle, curriculum-making bodies the world over should suddenly decide to include the study of physics along these broader lines in the offerings of all institutions of higher education, the result would be confusion and catastrophe. With a few exceptions, teachers are simply not prepared to exploit such an unprecedented educational opportunity. We should immediately be exposed as bankrupt in the very region where we could reasonably be expected to be superlatively solvent. We could not hide behind the alibi that this field belongs to historians and philosophers. Appreciation of the significance of the scientific method can be conveyed only by those who are steeped in it themselves. It would be very illuminating to consider how many of our historian and philosopher colleagues could be relied upon for an informed, discriminating and penetrating presentation of the place of science in the general intellectual enterprise. The average would not be high. The late Frederick Barry once wrote:[3]

> The ultimate establishment of more liberal elementary courses in science cannot be avoided. It is necessary to our purpose that the humanistic liberalization of scientific studies be powerfully advocated and actively encouraged and at once; for the obvious reason that we must depend on the scientists to devise our basic cultural courses in science.

[3] F. Barry, *The scientific habit of thought* (Columbia Univ. Press, 1927), p. 321.

Some teachers unfortunately take the position that the liberal approach should be segregated in courses about science, labeled history of science, or philosophy of science, preferably restricted to science majors as the only students who know enough subject matter to possess a foundation for generalization. This loses sight of the fact that it is the general *educated public*, nonscientific as well as scientific, that needs this message. Our principal access to this public lies in courses designed to meet the science requirement. The very establishment of the science requirement was, indeed, made on the assumption that this would be the kind of contribution that the sciences would make to general education, an assumption which we have woefully failed to justify.

Let us not delude ourselves into believing that we have no responsibility for the consequences of failing to provide the requisite breadth of science education. One of the consequences is that the sciences, and physics in particular as the most technological and highly specialized science, is in danger of losing even the small place it now holds in the liberal arts curriculum. The anticipated postwar boost of interest in physics simply has not materialized as far as liberal arts colleges are concerned. The prospect is, indeed, that, in the course of a century or less, physics at least will disappear as a significant feature of the liberal arts program. It is cold comfort to realize that with such disappearance will occur the eclipse of the liberal arts college, for it cannot remain liberal nor long exist in any significance after the loss of what is potentially its most important offering.

But more important still is the effect on social vision of confining our treatment of physics largely to its technological or even its purely scientific aspects. The great danger in our teaching is in the intellectual myopia that results from confining our teaching to the preparation of specialists. Not that we should *not* prepare specialists. We shall always have to do that and do it well. The danger lies not in doing that job too well but in doing it exclusively. Professor Sigerist, of Johns Hopkins University remarked shortly before the war:[4]

> If the German academic world surrendered so readily to reactionary forces, it was largely due to the fact that it consisted of men who were specialists and nothing else. If we wish to educate a citizen to be able to think in terms of science and a scientist prepared to participate in social action, we must change our teaching.

There is reason to fear that we ourselves are not immune to the malady that overtook the German academic world. Indeed, in some ways we are following closely the same path. Because of the narrowness of specialization characterizing our graduate training and because of the pressure in later professional experience toward developing a fertile and rapidly expanding field, we science teachers have become primarily subject-matter specialists, and only secondarily educators. In some cases our preoccupation has been with research; in others with the training of specialists, a very different undertaking from the problem of fitting one's subject into a matrix of general education. Unless and until we realize and discharge our proper function in this area, we shall bear a heavy share of the responsibility for the inevitable repetition of the social failure of the once highly vaunted German educational system.

This brings us to the second "requisite" of Dean Cooper's list, namely, "responsible citizenship." Dean Cooper defines the responsible citizen as one who is "prepared to participate as an active, responsible and informed citizen in the discussion and solution of the social, economic and political problems of American and international affairs." Do physicists typically make the contribution in this area that can reasonably be expected of them? Most of you would probably say, "Yes." I say most emphatically, "No." It may possibly be true that physicists are no more negligent than others about their duty at the polls, in supporting the Community Chest and responding to the numerous other appeals that come to all citizens. This is all very well, but for physicists in "the scientific era" it is not enough. Men of science in general, and physicists in particular, have had a much more far-reaching responsibility to society ever since the time of the industrial revolution.

Beginning about 200 years ago, physicists have been placing in the hands of society more and more powerful tools. These tools have been

[4] H. E. Sigerist, *Sci. and Society* 2, 3 (1938).

used to develop our natural resources and, in the process, to exploit human beings with scant regard for their basic civil liberties. We never hesitate to claim credit for the former, but we hasten to disclaim any responsibility for the latter. Sometimes indeed we feel irritated at those who try to make us see the implications of our science as its growth progressively affects human values.

Only in recent months have a few physicists exhibited any concern for the broad social effects of their science on the future of mankind. These are largely confined to men who have been directly involved in the development of atomic power for military purposes. With a hearty "Godspeed" to these prophets (for they are no less), the fact remains that the social effects of physics in the past have been fully as profound as those of the future about which these men are so deeply concerned. The development of steam power in the eighteenth century, of the internal-combustion engine and the electrical industry in the nineteenth, and of radio communication and atomic power in the twentieth—these are all parts of the same pattern.

In spite of all this, the typical physics teacher is still an intellectual isolationist of the deepest dye, usually without realizing it. His tacit philosophy is substantially that of Pierre Curie who, in 1894, recorded it in a letter to his fiancee, Marie Sklodowska. Discussing their prospective future together, a future that was to be so momentous to the scientific world, he referred to ". . . your dream for your country; our dream for humanity; our dream for science," and continued, "Of these dreams, I believe the last alone is legitimate. I mean to say by this that we are powerless to change the social order. Even if this were not true, we should not know what to do. And in working without understanding, we should never be sure that we were not doing more harm than good by retarding some inevitable evolution."

I believe this statement of Pierre Curie to be representative of the attitude of men of science from the beginning. With certain exceptions it is still representative today. Just prior to our entry into the last war it was repeated in a beautifully written essay entitled "The Wave of the Future"; and many men of science, among others, more than half agreed with Anne Lindberg that we should not attempt to influence a trend of the times. Perhaps, so the thought ran, certain undesirable features of this trend, however baleful they might appear to those who were suffering under Fascism and Nazism, were after all only the "scum on this Wave of the Future." Sir Josiah Stamp recently remarked that our disastrous failure to provide statesmen and administrators with some comprehension of the scientific and technical forces which are molding society has been matched in catastrophic consequences only by the complete and willful blindness of men of science to the social consequences of their work.

It required the shock of Hiroshima to open our eyes to what the attitude of Pierre Curie and his like (and who of us have not been "his like") was doing to the world. And even now we are only partly awake. Being forced to admit a considerable degree of responsibility for what is done with the new power we have unleashed, we still think of the present situation as somehow unique. We have not, to any considerable degree, reached the point where we can see that an equal degree of responsibility for the whole structure of modern society is involved in the very fact, which we admit and even proclaim freely enough in other connections, that we live in the scientific era. Yet until we do, we shall not be able even to comprehend, much less discharge, our peculiar civic responsibilities, which extend uniquely far beyond those of most other citizens.

It is profoundly to be hoped that we may not have to learn these responsibilities "the hard way," as many European and Japanese men of science have. The most revealing statement on this that has come to my notice is the address by Dr. Karl Jaspers on the occasion of the reopening of the Medical School of the University of Heidelberg in August 1945. He makes it tragically clear that the German downfall was basically caused by a long term wide disregard of the social responsibilities of science. I shall quote a few extracts from this address to show (as well as limited extracts separated from their context can show) Doctor Jaspers' development of this theme. It is a tragic document which all of us could profitably read in its entirety.[5]

[5] A translation appeared in *The American Scholar* for Spring 1946. The present translation, however, was made

We survivors managed—at a price—to avoid death. When our Jewish friends were marched off, we did not go into the streets to cry out until we too were annihilated. We preferred to remain alive with the weak but rational excuse that our death would have been of no avail. That we live is our guilt. We know before God our deep humiliation.

But it was our own decision that we should live. This decision implied that we accepted the consequences of existing under the given conditions. The only dignity left to us in our debasement was integrity and the will to work with infinite patience. Our paramount duty was to be deserving of the life for which we had been saved. . . .

The honesty and clarity necessary for this could be achieved only when the two pillars on which science rests were re-erected. These pillars were intellectual integrity and respect for the human being, or science and humanitarianism. These pillars had crumbled. Had they stood firmly from the beginning, the invasion of National Socialism into the field of science would have been impossible. . . .

I dare to maintain that much of what posed as scientific in our medical literature of the last fifty years was really unscientific because it was not written in the humanitarian spirit. This was what, above all other influences, opened the gates to National Socialism. The racial theory, for instance, contains certain elements which probably play a considerable role in the basic structure of man's existence. Yet the greater part of what was taught during all that time was pure swindle. The consequences were first a pernicious pseudoscientific myth which distorted all perception, and this was followed by all those criminal acts of extermination directed against the so-called inferior races. . . .

The other pillar of science is humanitarianism. It is the respect for man as man. Each individual is essentially infinite. No scientific concept can grasp him as a whole. . . . This is the image of man that had been lost. For that loss a large part of our scientific literature was in the first instance responsible. We have now bitterly learned our lesson. No true image of man without God. The full image of man must be regained.

by a neighbor of the writer, Mrs. Reche Jaszi, to whom grateful acknowledgment is hereby made.

The thesis which Doctor Jaspers developed out of Germany's tragic experience had been the central theme of Christian doctrine for 2000 years. It will be one of the ironies of history if humanity should finally learn that lesson as the price of merely avoiding extinction, instead of in the more creditable form of an expression of mankind's highest idealism. But one way or the other, that lesson must be learned, and learned soon or our culture will go the way that some on a smaller scale have gone before us.

We, as teachers of physics, have at least two lessons to learn. One is the "influence of basic scientific principles on the development of human thought and institutions." The other is that of "responsible citizenship" at the level of responsibility which our profession imposes. Can we learn those lessons in time? If we cannot, our culture will disappear, either gradually or explosively, and join in oblivion the previous cultures unable to adapt themselves to their times. If we can, we shall have established a foundation for Pasteur's famous statement, "I believe invincibly that science and peace will triumph over ignorance and war."

The Place of the Physical Sciences in General Education

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Comments on Some of the Requisites of General Education

THERE are two rather general comments which I would like to make about some of the requisites of a general education as drawn up by Dean Cooper,[1] and so ably discussed by him and Professors Roller[2] and Taylor.[3] An inspection of this list of requisites of general education seems to show that the various objectives which they suggest lie on different levels of attainability and value. This difference is, in part, caused by the fact that some of the objectives are in the nature of ultimate objectives while others have a more immediate character. For example, the development of a satisfactory philosophy of life is hardly the result of four years of university training. It is rather the outcome

[1] R. M. Cooper, *Am. J. Physics* **14**, 387 (1946).
[2] D. Roller, *Am. J. Physics* **14**, 390 (1946).
[3] L. W. Taylor, preceding article, p. 68.

of years of reflective thought and experience. On the other hand, the aim to promote literacy in a student by developing his powers of effective communication is more immediate and specific. It has a fair chance of being realized before the student ends his formal education.

A considerable part of the confusion and controversy over educational objectives seems to arise from the failure to discriminate between ultimate and immediate objectives.

The process of realizing any objective of the ultimate type is seldom a simple one. It is more likely to be a complex act requiring many steps. For this reason it is frequently necessary to set up a sequence of intermediate objectives to serve as guide posts. Progress toward the final goal then consists in focusing one's attention and energy on the realization of the immediate objective ahead, without, however, losing sight of the ultimate goal. This is not an easy thing to do. We are inclined either to make a fetish out of the immediate objective while forgetting the ultimate goal, or to spend our energies fruitlessly on something remote by not attending to the business at hand.

But there is a second and perhaps more important factor which explains the varying character of the objectives. Dean Cooper has already mentioned this factor as involving two different approaches to the needs of general education, the one functional and applicational, and the other intellectual and disciplinary. There seems to be a fundamental schism between the approaches to the objectives of general education, which may be traced back to two different philosophies of life. Morris Cohen[4] expresses these two different points of view by means of thesis and antithesis.

> [*Thesis*] Wisdom means rationally managing or organizing our activities so as to achieve a maximum of attainable ends or goods of life.
>
> [*Antithesis*] Men can achieve their noblest character only by pursuing some great unattainable goal, or by subordinating themselves completely to some cause bigger than themselves. The pursuit of merely natural or worldly satisfactions will not of itself give human life sufficient scope, dignity, and real worth.

The author goes on to say:

> The thesis seems almost self-evident and needs no supporting argument. . . . But the upholders of the antithesis are not without true insight. For by ignoring the unattainable beyond us, we cut ourselves off from that intense enthusiasm which brings out our utmost efforts and prevents our existence from becoming drab, our pleasures unimaginative, and our practical activities narrowly mechanical.

These two antithetic points of view concerning a philosophy of life furnish much of the background for the controversy over educational objectives and cut across the whole field of general education. They determine, to a large extent, the different kinds of educational objectives and curriculums that flood the educational market places at the present time and that range from St. John's "classical" curriculum to the "functional" curriculum of the General College at the University of Minnesota. It is worth noting that in this range the physical sciences are not left out even though they may not occupy a dominant position. Hence it would appear that these sciences can and ought to make some contribution in the field of general education.

The Physical Sciences and General Education

A good course in physics, or in the physical sciences, may contribute to practically all of the requisites of a general education given by Dean Cooper. For example, there is no field of study, with the possible exception of pure logic and mathematics, in which it is so important to be careful about the choice of words and sentences in order to communicate effectively ideas and information. Also, plain honesty and objectivity play important roles in physics, as well as in good citizenship. But the primary contributions which physics makes to general education flow from the specific character of physics and the physical sciences.

It would be carrying coals to Newcastle to spend much time here in describing the specific character of the physical sciences. These sciences, as you well know, are concerned with the examination of rather highly restricted aspects of nature. The emphasis is on measurement and experimentation. Logic and critical thought play important roles in analysis, and creative imagination is the *sine qua non* in the development of theory. But in all events the final court of appeal is an experimental result or an apparent fact. When fact and theory fall out, it is the theory that must be modified.

[4] Cohen, *Reason and nature* (Harcourt, Brace, 1931), p. 445.

These are a few of the more important elements of the method used in the physical sciences. This method has been so successful and powerful in the field of the natural sciences that many are inclined to assume that it ought to work equally well in all fields of knowledge. I am inclined to be somewhat skeptical of that assumption. The methods used in studying any field of knowledge depend largely upon the material being studied and the kind of knowledge that is wanted. Hence there is no guarantee that a method which works well in physics will without drastic modification work equally well in philosophy or, more practically, in running the government or in bringing up a family. Hence one of the important aims of a general education physics course should be to show the connection between method and subject matter in physics, and to point out the limitations of that method when applied to foreign material.

It is for this reason that a study of the physical sciences, for example, cannot replace a study of the social sciences, and *vice versa*. It is easy for the physical scientists to agree with the statement:[5]

> The social studies cannot compete with pure mathematics and the natural sciences in exhaustive analysis, rigorous inference, and verifiable interpretations. Their methods are by nature such as to forbid the substitution of these studies for the more precise and established disciplines. The latter must continue to supply a distinctive and fundamental type of exercise in consistent reasoning and fidelity to empirical data.

But Aristotle's[6] warning on this point is also worth repeating:

> Discussion . . . will be adequate if it has as much clearness as the subject matter admits of, for precision is not to be sought for alike in all discussions. . . . It is the mark of an educated man to look for precision in each class of things just so far as the nature of the subject admits; it is evidently equally foolish to accept probable reasoning from a mathematician and to demand from a rhetorician scientific proofs.

It is clear, however, that at the present time, and in most instances, traditional courses in physics and in the physical sciences are not making the contribution they should be making in the field of general education. Before this can occur there will probably have to be not only a change in subject matter and teaching methods but also a shift of emphasis.

Some Important Components of a Course in Physics Designed for General Education

The celebrated French philosopher, Henri Bergson, points out in his *Introduction to Metaphysics* that there are two fundamentally different ways of knowing a thing: it may be known from the outside as a spectator, or from the inside as a participant.

In designing a physics course in general education the first question that has to be answered is which of these two different ways of knowing physics is to be chosen. Is the course to be *about* physics or *in* physics? If it is the former, the task will be easy and the results, I suspect, meager. If it is the latter, the task will be formidable, and there will be no guarantee of good results.

It is difficult to see how any course about physics or the physical sciences can make an essential contribution to general education. True it is, that one does not have to worry with mathematics and laboratory work in such a course. Unfortunately, as the Harvard report[7] points out:

> What might be conveyed without them is not only not science, but is in a very real sense antiscientific. It comes perilously close in spirit to scholasticism. It possesses the typically scholastic reliance upon verbal authority—in this case the authority of the writer of scientific texts—it has the predominantly deductive logical structure, and the same preoccupation with words, rather than with objects and processes which they only imperfectly symbolize. The thought that an understanding of science might be conveyed as well or better without direct observation, experiment, and mathematical reasoning involves a fundamental misapprehension of the nature of science.

On the other hand, the choice of a course *in* physics poses the very real problem of the part that mathematics and laboratory work should play in the course. These are real barriers for many students, but to avoid them is to lose the very values which the course should contribute to general education.

The role that mathematics plays in physics

[5] T. M. Greene, *Liberal education re-examined* (Harper, 1943), p. 56.

[6] W. D. Ross, *The works of Aristotle* (Oxford Univ. Press, 1925), Vol. IX, 1094b.

[7] *General education in a free society* (Harvard Univ. Press, 1945), p. 153.

is extensible. It may be limited, but it should not be eliminated. A great deal can be done with arithmetic, elementary algebra and plane geometry. And the student's propensity to use proportions should certainly be capitalized on.

That laboratory work should play an important part in a general education physics course is hardly debatable from our point of view, even though its exact nature may be open to question. It has been argued that the demonstration lecture can, if need be, replace laboratory work. While there may be a strong economic reason for this position, let us not make virtue out of necessity. In addition to the argument of economy there is another which may be summed up in the question: What difference does it make whether the student or the lecturer "throws the switch" in an experiment? Certainly there is none as far as the apparatus is concerned. But as for the student there is often considerable difference. In the former case he is participating in an enterprise, whereas in the latter case he is usually just a spectator often so far removed from the main event that he has to use a pair of binoculars to see what is going on.

This difference in point of view of the student is fundamental. If he is to get meaning and value out of the course, he must enter into the enterprise as a participant. Admittedly, the demonstration lecture might be modified in such manner as to encourage this sense of participation, and laboratory work does not necessarily guarantee it. But the chances of securing this condition are greater in a small laboratory section than in a large demonstration lecture.

One must admit that laboratory work, as it is commonly performed in the traditional physics course, has many faults. It is frequently regarded as a minor appendage or an afterthought of the course. And many of its weaknesses arise from this viewpoint. This state of affairs is unfortunate for all students of physics but it is especially serious for the student in a general education physics course. Engineering, premedical and other students who expect to continue in scientific work will certainly get plenty of laboratory work in their educational careers, and hence will have ample opportunity to participate in this aspect of the scientific method. But for the student whose course in physics or in the physical sciences is likely to be a terminal course, the situation is different. If he does not realize the place of experiment in physics at the time, he may never get another chance.

There is another argument for laboratory work that deserves mention. Even in a beginning course, physics can be, and often is, a very abstract and theoretical affair. Symbols, definitions and equations abound. The instructor, with his backlog of experience, is not likely to confuse the symbol with the thing it represents; the inexperienced student may very well do so. Hence, it is extremely important for the student that as many symbols as possible be "thingified," even at the expense of cutting subject matter. This can usually be done best in the laboratory where the student actually handles resistances, meters, spectrometers, lenses and so forth.

Although there are urgent arguments for making laboratory work an integral part of a course in physics designed for general education, it is also clear that the traditional type of work in the laboratory will hardly suffice. Much reflection and experimentation may be necessary before the question, "What kind of laboratory work?," can be answered satisfactorily. But its general character is fairly clear:

> It should be planned to illustrate the methods by which physical problems are approached and solved. Every effort should be made to convey these as genuine experiences, either by presenting the student with problems of which he does not know the answer or, when this is impracticable, by casting back the situation into the historical framework in which it constituted a genuine issue. The student should thus have a series of real experiences in the scientific solution of material problems.[8]

The solution of physical problems usually demands quantitative work. Furthermore, physical laws and principles are quantitative in character, and cannot be exemplified by purely qualitative experiments. Hence, it would be a mistake to underemphasize the quantitative nature of the work in the laboratory. But a quantitative result is of little value without some knowledge of the error involved in it. Thus some simple treatment of errors would seem to be essential in the laboratory.

A third component in a course in physics

[8] Reference 7, p. 228.

designed for general education is the role that relations between physics and other areas of knowledge should play. It is an important role and should not be neglected. The excuse that there is no time left for such incidentals because there is so much subject matter to cover misses the point entirely. These relations are not incidentals in this course but essential components of it. Without this element the course can hardly be considered satisfactory from the standpoint of general education. The real difficulty of achieving this end often lies in the generally faulty educational training of physics teachers. Many of them have been educated on too narrow an educational base. They view physics from the inside, and from the inside only. What is needed is a kind of transformation that reverses the traditional positions of instructor and student in physics. At certain times during the course the instructor should look at physics from the outside while the student is viewing it from the inside.

Condensed Version of Discussion Aroused by the Four Papers on General Education Read at the Iowa Colloquium of College Physicists

C. J. Lapp, *State University of Iowa.*—On the basis of 15 years of experience and experimentation with physics in general education, I would say that Professor Wall's statement that the course must not be "about physics" but must "be physics" is entirely correct. With Professor Roller I would agree when he says that the material must be selected; indeed, it must be highly selected. He also said that it must be analytic and not descriptive; I would say, it must be *both* analytic and descriptive.

Dean Cooper's point concerning transfer of training is exceedingly important. Transfer from one area to another is unimaginably difficult. I find that after one teaches problems in terms of apples one day, the students may not be able to solve problems in terms of peaches the next. Transfer must be constantly emphasized if values of physics are to be taken into other fields of life.

Effective communication should receive great emphasis, especially the use of curves, charts, diagrams, symbols and the language of variation and proportion. I do not say equations, because in such a course it would not be possible to use many equations.

E. W. Skinner, *Northwestern University.*—Professor Wall appeared to be of the opinion that laboratory technics are not necessary for premedical and predental students since such students will be taught these technics at a later date. Actually, we teachers in the professional schools base our laboratory technics on those that we assume the student has learned in his work in the liberal arts college. It seems to me that I can always pick out those students in my classes who have had a first-class course in physics, including rigorous laboratory work. I grant that it is important to inspire the student to acquire the qualities listed by this symposium, but we must not forget that we are teaching physics and that our primary objective is to teach the student fundamentals.

Concerning the judicious use of laboratory time, as suggested by Professor Roller, I suggest that physics teachers try the experiment that we have been carrying on. We have written a laboratory manual which assumes that the laboratory work will be done before the theory is taken up in the classroom. In other words, we make our experiments really experimental in the strict sense of the word. We feel that the laboratory thus becomes a place where information is gained for the first time and that the class discussion so far as possible emanates from this experimental material.

L. A. Turner, *State University of Iowa.*—I should like to amplify a point, made by Professor Roller, that physics is a very simple subject. Students may not agree with this, but it is nevertheless true. We deal with things that are very definite. Some of it, to be sure, is subtle, but that is not what I am talking about here. We have high standards that seem to the student to be most exacting ones. That is in the nature of the subject. We must bear it in mind in dealing with the students kindly, but we must teach them that there are subjects of this kind as well as other subjects that are less precise.

With regard to effective communication, I agree that we can contribute something. Professor Lapp forgot to mention the English language. I have been in the past few months helping to edit some books. The chapters have been written by men who are very able in their fields, but their writing is not a credit to their English teachers or to their colleges. I am not suggesting that we become an adjunct to the English department, but when dealing with material that can be expressed clearly we can show students how language can be used to better advantage to express clear thought. That is certainly part of our job.

W. P. Gilbert, *Lawrence College.*—Physics is simple, but it is because of a process of isolation. On the second point, with regard to communication, I agree that we should emphasize English in physics. We should make ourselves an adjunct to the English department and should see that our students write clearly. We must not accept garbled statements because they contain the facts of physics. A. N. Whitehead has said "Science repudiates philosophy." The difficulty lies in us, the teachers. We are too willing to let the philosophers teach the philosophy of science and the historians teach the history of our subject. We should accept these responsibilities ourselves.

E. Bollhoefer, graduate student, *State University of Iowa.*—(i) Concerning effective communication, we introduce the student to numerous words that he already knows, such as *work* and *force*, but of which he has had no precise definition. We do give him good definitions, but often he retains his own inaccurate and inadequate notions; he must be made to understand that these words have precise meanings apart from everyday usage.

(ii) I find that students have not been given a feeling by the classroom instructor that the laboratory work is important—that physics is a practical, experimental science. They think that laboratory work is merely something added to the course rather than that it is the whole foundation to the facts of the course. It is impossible in one short laboratory period a week for the laboratory instructor to change this impression that has been gained in the classroom.

W. Noll, *Berea College.*—I want to raise a question of fact regarding Sigerist's statement that the German academic world surrendered readily. I understand that inmates of the infamous concentration camps were mainly Germans and presumably people who did not surrender readily or at all. I discussed this matter with our philosophy professor who gave me two references: (i) the educated group capitulated to Hitler because they had been trained to give iron-rod obedience to the State, not because they were specialists (J. Dewey, *German philosophy and politics*); (ii) the lower middle class gave strength and devotion to Hitler because they were abandoned by outside neighbors—France and England (E. Fromm, *Escape from freedom*).

D. Roller, *Wabash College.*—There is some loose talk going the rounds about the dangers of specialization, the implication sometimes being that one who has specialized highly in some particular field will necessarily be a person of narrow education. More emphasis should be given to the idea that a broad education will be most effective when accompanied by profound knowledge and great skill in some one area; the "man of one book" has certain tremendous advantages over the tyro in everything.

A person with a truly profound knowledge of a major field, such as physics, is almost sure to be broadly educated; and if he is to be most creative in research or most successful in teaching, he will also have many of the personal and social qualities that are commonly regarded as important outcomes of general education. We need a better concept of "intelligent specialization."

J. C. Jensen, *Nebraska Wesleyan University.*—Concerning effective communication, I have for many years cooperated with the English department to the extent that all laboratory reports are graded on "form," "neatness" and "accuracy." "Form" includes the student's discussion of the theory of the experiment in his own words, not merely a copy of the treatment given in the laboratory manual.

Now that the war is over, I hope to offer again a course in the history of physics for upper-division students in which the philosophic aspects of the subject will be given special emphasis.

J. A. Eldridge, *State University of Iowa.*—More time should be alotted to physics in the general education curriculum. The typical curriculum allows for a half-year of physics in the form of a survey course. Dean Cooper has itemized the essentials of a liberal education. When one considers how many of these are related to physics, this half-year seems to be inadequate.

It is a mistake to make physics one part of a survey course. The subject is already a survey of many fields. I wonder why we do not have survey courses in languages—all the modern languages in one year. Judged by Dean Cooper's essentials, a foreign language does not contribute to a liberal education as does a science. But no one seriously suggests a survey course in languages. We all feel that a student should learn a language, not learn about languages. Is not much the same true in science? If we are to teach the scientific method, is it not better to study some one science relatively intensively rather than a smattering of sciences?

H. K. Schilling, *Pennsylvania State College.*—As I look back over my not too many years of teaching experience, it seems to me that in such meetings as this everybody is always interested in the important topic of *objectives*, but that after the meeting everybody goes home and tends to his knitting in his customary way. At the next meeting a new cycle begins, but in the end nothing much is accomplished. I wish the good Lord would raise up a Moses among us to lead us out of the wilderness of present-day physics teaching.

We have never, as a profession, made a serious attempt to define the specific objectives of physics teaching. We simply "teach physics," that is, pass out conventional subject matter. While we are interested in the objectives that have been proposed here today, do we know how to implement them or to put them to practical use? In general, our textbooks are not written with a view to achieving such objectives.

I wonder whether our Association of Physics Teachers, which has done so fine a job in other ways, might not help us solve this problem by creating an agency that would direct, or at least sponsor and encourage, cooperative educational research and experimentation in teaching. Cooperative research in other fields was a great success during the war. Now that peace has come, might it not be possible to achieve some kind of cooperation in the field of teaching? The Association might appoint a committee on educational research that would outline, say, a five-year plan for a cooperative study of possible methods by which the first three objectives proposed today (or others) might be realized. Thereupon, at the call of the committee, teachers interested in such an investigation would get together and agree on details. Later other objectives would be attacked and other methods tried. The committee would act as a clearing house for reports and proposals as the experiment progressed and would arrange for annual meetings of the collaborating group. In time some such procedure might accomplish a good deal.

L. W. Taylor, *Oberlin College.*—I am reminded of the mother who took her little son to task for abusing his sister. She said, "Now I'm sure you're not really that bad; Satan must have tempted you to do that." His answer was, "Well, Satan may have tempted me to pull her hair, but kicking her shins was my own idea." If you have found inspiration

in this discussion of how physics can be made to meet some of the requisites of general education, it is partly because, besides being receptive to suggestion, you have contributed your own ideas.

One thing requires emphasis, however. Let us not allow the conservatism which is all too widespread in our profession to prevent our accepting new ideas and putting them into practice. This is a point on which I have learned to be profoundly pessimistic. I was commenting to a colleague just this morning, for example, on the way we physicists have disregarded a major recommendation of our own duly authorized standardizing body. In 1935 the International Committee on Weights and Measures legislated that "the actual substitution of the absolute system of electrical units (the "MKS system") shall take place on January 1, 1940." It is now 1946 and next to nothing has been done to put this sorely needed reform of our units into effect. If we physics teachers hope ever to improve our own procedures, we shall have to become more cooperative than that.

But a merely cooperative attitude is not enough. If we are to improve our instruction in the ways that have been suggested here, some of us will have to broaden our own horizons first. Perhaps the problem will best be described by another story. A passer-by, commenting to a small boy on what a smart dog he had, asked how it had been trained. The boy said: "It's this way, lady; you just have to know more than the dog does." Our own horizons are in many cases more restricted than we ourselves realize. Our students are often more broadly trained than we are. If we are to be effective guides in broadening the instruction in our own subjects, most of us will have to go out of our way to "know more than the dog does."

R. Cooper, *University of Minnesota*.—Professor Stewart has suggested that I offer a benediction. I have taken rather careful notes, but am not sure that they are of any value. However, I might summarize what has been said. Three or four things seemed to be highlighted in the panel and the discussion. One is the emphasis everyone placed on communication, upon symbols and better use of the English language itself; it is the one suggestion that came up again and again, and is obviously important to the students. Professor Taylor emphasized the social implications of physics—the fact that the instructor must make the student see the connection with social phenomena and that there be proper transfer.

Also emphasized was the development of the scientific method and its application to all kinds of phenomena. I have been a little disappointed that there has not been more emphasis on the methods by which we may develop critical judgment in the laboratory and elsewhere. I am quite certain that it did not happen when I took physics. I took laboratory by the cookbook method. I was told exactly what to do and I did it, and that was all there was to it. But I did not learn to face problems with intellectual discipline.

Physics teaching has probably improved a good deal since I was a student, but in all fields we need to emphasize this discipline. We need to show that these suggestions have a practical application. It is more difficult to do the thing than it is to sit here and talk about it. In our North Central Association studies we have been much concerned about implementation. Dean McGrath is launching soon a *Journal of General Education*, where there may be an opportunity to report when a person does some significant work in adapting college courses to meet life needs. Physics has done a wonderful job in creating technicians, but it has been deficient in teaching people how to translate these procedures into values that the average man can use. That is one of the compelling needs of our day, and now is the time to go out and try our hands at it. It will take imagination and courage. Experiments with procedures are definitely needed. Physicists here can adapt themselves and give leadership to other physicists and to other fields of learning.

New Members of the Association

The following persons have been made members or junior members of the American Association of Physics Teachers since the publication of the preceding list of new members [*Am. J. Physics* **14**, 444 (1946)].

Augustus, Sister M., IHM, Marywood College, Scranton, Pa.
Azbell, William, 200 S. Douglas, Peoria, Ill.
Bowles, Comdr. R. P., USNR, U. S. Naval Academy, Annapolis, Md.
Boyd, Bert B., 1010 N. College Ave., Natchitoches, La.
Bramhall, Ervin H., 3758 Manini Way, Honolulu 17, T. H.
Byers, Walter H., University of Oklahoma, Norman, Okla.
Copperman, Ruben (*J*), 2525 W. Columbia Ave., Philadelphia, Pa.
Dempster, Richard R., 505 N. 3rd St., Corvallis, Ore.
Ealy, Eleanor (*J*), Marywood College, Scranton, Pa.
Ferguson, Jeremiah M., 259 Main St., Hornell, N. Y.
Graves, Leon F., University of Houston, Houston 4, Tex.
Hahn, Thomas M. Jr., 9614 51st Ave., Berwyn, Md.
Hayes, John L., Lew Wallace School, Gary, Ind.
Keepin, George R. Jr., (*J*), Massachusetts Institute of Technology, Cambridge 39, Mass.
Kirsch, Simon C., Woodstock College, Woodstock, Md.
Kolstad, George A., Yale University, New Haven, Conn.
Lothery, Thomas E. Jr., P. O. Box 674, Lexington, Va.
Matthews, David J., Birmingham-Southern College, Birmingham, Ala.
Miller, Robert J. (*J*), 123 Forest St., Oberlin, Ohio.
Miner, Thomas D., 201 Kilburn Rd., Garden City, N. Y.
North, H. M., 7709 Adrian, Houston, Tex.
Olsen, Marvin, Wells College, Aurora, N. Y.
Powers, Philip N., 2703 Lee Blvd., Arlington, Va.
Rhodes, Jane Lockett, 545 S. 55th Pl., Birmingham 6, Ala.
Rinker, Jacob A., Eureka College, Eureka, Ill.
Sandy, Deniro D., 3334 W. Roosevelt Rd., Chicago, Ill.
Shatler, Albert V., 1003 North St., Newberg, Ore.
Simpson, Kenneth M., Santa Barbara College of University of California, Santa Barbara, Calif.
Smith, Harold E., 58 Argyle Rd., Milford, Conn.
Trumble, Lt. Comdr. Robert E. Jr., U. S. Naval Academy, Annapolis, Md.
Williams, E. Allen, Santa Barbara College of University of California, Santa Barbara, Calif.
Witz, Harold, 749 S. Kostner Ave., Chicago, Ill.
Wright, Byron T., University of California, Los Angeles 24, Calif.

Aim of Laboratory Experiments for Liberal Arts Students

ERIC M. ROGERS
Princeton University, Princeton, New Jersey

WHAT does the student think is the aim of his laboratory experiments? What does he expect to gain? What do his instructors expect him to gain? What do we hope he will retain as a lifelong contribution to his education? For many liberal arts students, temporary gains of preparation for a further physics course are nonsense, because for these students there is not going to be a further physics course.

In an earlier article[1] I made a plea for re-examining the content of courses and our attitude towards them, where we give them to students as their only physics course—often their only science course. If we ask what gains the student will carry to other studies and what he will retain ten years hence, we may feel tempted to throw out topics wholesale, keeping a few samples—thus gaining time to treat them carefully—to show the student what physics is like and how physicists work, and to give him some understanding of the nature of science and the scientific approach to problems. The student's own work in the laboratory may be the most important of all these samples. We shall not give him lasting gains by drill in scientific method nor by expounding fact and theory in lecture, unless we give him in the laboratory something nearer to a genuine sample of scientific work and help him thereby to feel that this strange subject is more real.

Yet actual laboratory courses often fail—at least in spirit. I go into a freshman laboratory and find a student wandering across to a neighbor with the question, "George, what do we put in this column?". We should not blame the student. We should blame ourselves and our hurry to get on to the next experiment. I meet another boy asking an instructor, "Will 0.286 *do* for the specific heat?" And another complaining, "My experiment is *wrong*. It gives 4.6 but *the book says* 7.2."

Behind all this headache and fog lie one or both of two great difficulties. First, the student does not understand clearly what he is aiming at; second, he is often bored with the thing he is supposed to be measuring. Looking back on our own first experiments in a physics laboratory, most of us remember such puzzles and boredom. Some students like a dull experiment that yields the "right" answer, but it is bad for their morals.

Let us re-examine the aims and content of our laboratory course with the ultimate good of general students in mind. Many technics can be omitted because they are not used much in this course, will not be needed for future courses and give only formal training unlikely to be transferred to life in general. Old friend though he is, the vernier can go. So can the Post Office box, parallax pins in optics, and the chemical balance. Other technics should stay and grow in importance—for example, graph-plotting and anything that helps develop a general delight in accurate measurement and a sensible treatment of errors. We may omit measurements of physical quantities that are of little immediate interest or use. As a prime example I mention an old victim, the measurement of specific heat by the method of mixtures. The method is messy and out of date by half a century, and specific heat is not as important to the student as old custom makes it seem; it does not "explain" sea breezes or make the student a better householder with an aluminum kettle. Latent heat and humidity are much more important. As further victims for omission I suggest: meticulous measurement of specific resistance, Young's modulus, coefficient of friction and focal lengths. There are good things concealed in these victims. A general investigation of friction can be a fine "sample." So can the stretching of a wire beyond its elastic limit. Experiments that aim at "verifying laws" need at least a change of title. The term "verify law" in an elementary laboratory experiment is a stricture on scientific attitude and makes the experiment a mockery of scientific work. Yet to follow some early experimenter's discovering of a law or to *test* a familiar law may be an admirable sample.

[1] E. M. Rogers, "Samples *versus* survey in physics courses for liberal arts students," *Am. J. Physics* **14**, 384 (1946).

If in our teaching we want to give students such things as an understanding of physics, a respect for honest, accurate investigation, a ruthless abiding by our own experimental results and a delight in clearing up relationships, then I suggest the following kinds of experiment for the laboratory.

(1) *Investigations*.—The standard example here is the investigation of the simple pendulum. We must be honest and admit it has been done before, but we may point out to the student the pleasure of trying the experiment for oneself and the value of adding one's own witness to the growing body of evidence. We can most of us devise more of these experiments, and we must not be deterred by finding that many of them are largely qualitative. I suggest: the linear motion of a wheel rolling down rails (leading to graphs); a cooling curve from liquid to solid; electromagnetic induction; Ohm's law; and such qualitative things as an investigation of conduction, convection and radiation.

As one of the last-mentioned experiments, I have students hold a test tube full of water at the top and heat it at the bottom; then repeat the experiment, heating it at the top and holding at the bottom. High-school students think this undignified, but undergraduates are surprised at the amount of information they can deduce. A good student will say, "I note that glass is as poor a conductor as water." One student watching specks of dye placed in the water said, "I think the mean free path of dye molecules must be comparable with that of warm water molecules." His evidence was scanty, but his attitude was good.

In such investigations we want fewer cookbook instructions; only a few suggestions from the instructor, but much encouragement.

(2) *Clarification experiments*.—Atwood's machine is a poor way of measuring g but can be illuminating when we discuss the nature of mass. Making an ammeter from a galvanometer clears up a lot. Making an astronomical telescope is a delight and leads of its own accord to measurements of magnifying power.

(3) *"Grand" experiments*.—Given time, a student enjoys an occasional complicated experiment; for example, a measurement of e/m or a measurement of wave-length by Newton's rings —harder but far more thrilling than a determination of focal length. Even a force-table experiment can be complicated and satisfying. We should, however, beware of experiments that provide data mysteriously ready made, such as, "the mass of the wheel is given you as 426 gm."

In all laboratory experiments we need time to explain the aim and discuss it, then re-explain it (the aim, not the experiment) half-way through the period, and discuss it again next week. And we need time for the student to mull over the aims and technics of the experiment. If he needs to continue next period, it may be wise to let him. Sometimes a complete repetition of the experiment will greatly benefit his understanding and self-respect. We ask him, "How long would this take to do again? How much better could you do it?" If he is keen and delighted we may build much by letting him repeat the experiment—if only we can remove the incentive and threat of grades.

We need to make of our laboratory, through our attitude and the student's, a place of scientific work—not an intellectual housing slum. What matters most is not what experiments we choose but what aims we put before the students. The aims in many laboratories and many books now are poor ones. The student does not want to measure the value of specific heat accurately. He is surprised at being asked to verify Hooke's law because he thought it was well known already. His aim is fuzzy (often through lack of time in the laboratory) and it is frequently poor, even down to the level of "to please the teacher." If we choose fewer experiments—to gain time for discussion and more careful work; for repeating experiments *when the student wants to*—we can present many of the experiments we now use with a better aim and with hopes of better answers to the key questions: "What do we hope the student will gain, as part of his real education? What does the student think is the object of his experiments?".

Discontent is the first step in the progress of a man or nation.—OSCAR WILDE.

Reproduction of Prints, Drawings and Paintings of Interest in the History of Physics

29. Steam Locomotion as an Art Subject

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ONLY rarely have either great artists or great poets dealt with steam locomotion. This is perhaps understandable even though it is regrettable, for as the Scottish engineer says in KIPLING's "M'Andrew's Hymn":

Lord, send a man like Robbie Burns to sing the Song o' Steam!
To match wi' Scotia's noblest speech yon orchestra sublime
Whaurto—uplifted like the Just—the tail-rods mark the time.
The crank-throws give the double-bass, the feed-pump sobs an' heaves,
An' now the main eccentrics start their quarrel on the sheaves:
Her time, her own appointed time, the rocking link-head bides,
Till—hear that note?—the rod's return whings glimmerin' through the guides.
They're all awa! True beat, full power, the clangin' chorus goes
Clear to the tunnel where they sit, my purrin' dynamoes.
Interdependence absolute, foreseen, ordained, decreed,
To work, Ye'll note, at any tilt an' every rate o' speed.
Fra skylight-lift to furnace-bars, backed, bolted, braced an' stayed.
An' singing like the Mornin' Stars for joy that they are made;
While, out o' touch o' vanity, the sweatin' thrust-block says:
"Not unto us the praise, or man—not unto us the praise!"
Now, a' together, hear them lift their lesson—theirs an' mine:
"Law, Orrder, Duty an' Restraint, Obedience, Discipline!"
Mill, forge an' try-pit taught them that when roarin' they arose,
An' whiles I wonder if a soul was gied them wi' the blows.

PLATE 1. Rain, steam and speed: The Great Western Railway. [From the painting by J. M. W. Turner in the National Gallery, London.]

PLATE 2. The first railway train in the State of New York, 1831. [From a colored lithograph after the painting by E. L. Henry.]

In 1844, however, that master of light and color, J. M. W. TURNER (1775–1851), was inspired by a scene on the Great Western Railway in England to paint a superb picture dedicated to steam and to speed. Although an old man at the time, TURNER comprehended the dynamic poetry of a train in motion through a landscape simultaneously swept with rain and drenched with sunlight, and portrayed it upon a canvas that now hangs in the National Gallery in London. This beautiful painting is here reproduced in black and white (Plate 1), even though it is impossible to convey the true feeling of a Turner without color.

While no other treatment of steam locomotion compares in artistic merit with TURNER's "Rain, steam and speed," there do exist a few paintings of early trains that merit reproduction in a series of this kind because of their historical accuracy. Among these are some of the works of the American historical painter, EDWARD LAMSON HENRY (1841–1919). To quote from the *Dictionary of American Biography:*

> Henry's major interest was in the past life and customs of the United States, especially during the first half of the nineteenth century. He began soon after his return [from study in Paris under Suisse, Gleyre and Courbet] to paint pictures which were accurate to the last chair and the most minute button. Owing in part to his attention to detail, his work was of greater historic than artistic merit. . . . Primarily an illustrator in oils, he found an appreciative public in that vast majority which demands of a picture first of all that it tell a story.

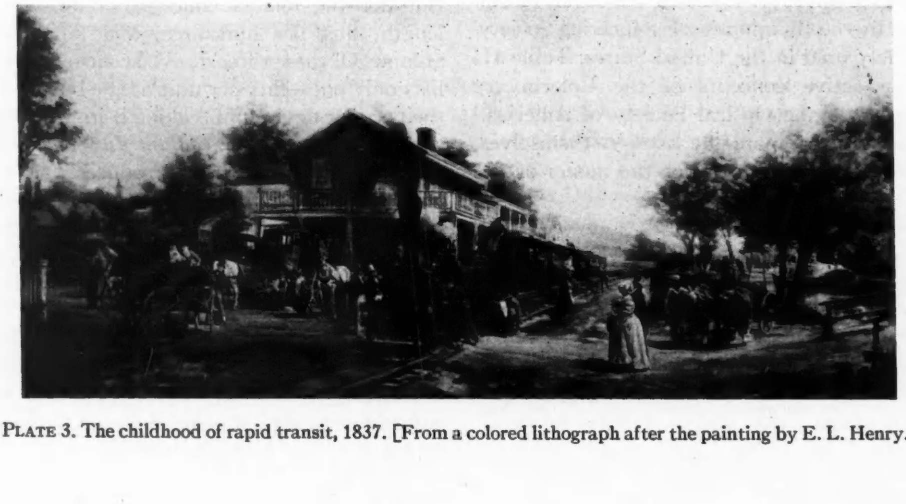

PLATE 3. The childhood of rapid transit, 1837. [From a colored lithograph after the painting by E. L. Henry.]

Plate 2 reproduces HENRY's painting of the first train operated in the state of New York. The locomotive portrayed, named the "De Witt Clinton," made its first trip in July 1831, over the Mohawk and Hudson Railroad (now the New York Central). On August 9 it made the trip from Albany to Schenectady, a distance of 17 mi, in less than one hour.

A painting of a railway scene of a somewhat later period (1837) by the same artist is reproduced in Plate 3. Unfortunately, a copy of HENRY's better-known painting, "Railway Station—New England" is not available to the writer. Although not great art, HENRY's paintings, because of "their rare sincerity and their quaintness," probably "will always be of interest and of value . . . and will throw an ever-penetrating light into our vanished customs and past social history."[1]

[1] Lucia Fairchild Fuller, *Scribner's Magazine* **66**, 256 (1920).

Internationality in the Names of Scientific Concepts: Comparison of Photometric Systems

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and

DOMINA EBERLE SPENCER, *Tufts College, Medford, Massachusetts*

IN a previous paper[1] a complete set of concepts was developed for radiometry, photometry and colorimetry, with international names for each concept. This example showed that the general principles previously proposed[2] could be applied to a specific problem. The active concepts, with their English names, are given again in Table I; and the purpose of the following paragraphs is to compare the proposed international system with three systems of nomenclature that are now in use.

Table II gives the active concepts recommended by the Illuminating Engineering Society[3] and widely used in the United States. Table III lists the active concepts of the Colorimetry Committee of the Optical Society of America.[4] We shall first compare the *concepts* themselves, after which we shall consider the *names* of the concepts.

[1] "Internationality in the names of scientific concepts: proposed international photometric system," *Am. J. Physics* **14**, 431 (1946).

[2] "Internationality in the names of scientific concepts: a method of naming concepts," *Am. J. Physics* **14**, 285 (1946).

[3] "Illuminating engineering nomenclature and photometric standards," *Illum. Eng.* **36**, 815 (1941).

[4] "Report of the Colorimetry Committee," *J. Opt. Soc. Am.* **34**, 246 (1944).

1. Comparison of the Concepts

The number of concepts is, to a great extent, arbitrary. If any one quantity in Table II (say, *radiant flux*) is taken as fundamental, then every other concept in the radiometric column is determined by definite spacial and temporal relations, and every concept in the photometric column is obtained by evaluation of the corresponding radiometric quantity with respect to the standard lamprosity function $v(\lambda)$.

Starting with *radiant flux* in Table II, one may consider the related concepts of flux per unit length, flux per unit area, and flux per unit volume. Of these three derived concepts, Table II lists only one—flux per unit area—but resorts to the peculiar device of breaking it into three parts with different names: *radiant flux density*, *radiancy*, and *irradiancy*. This introduces unnecessary complexity, since all three have the same dimensions and should be considered as one concept. In the photometric column of Table II, on the other hand, only one concept is listed in this group, but it is the wrong one. The proposed system of Table I employs only one flux density (D, *pharosage*) in accordance with the principle of obtaining maximum simplicity. If it is necessary to distinguish pharosage *from* a surface and

TABLE I. Proposed international concepts.

Radiometric				Photometric			
Symbol	Dimensions	Unit	English name	Symbol	Dimensions	Unit	English name
F_r	$[P]$	watt	radiant pharos	F_l	$[F]$	lumen	luminous pharos
D_r	$[PL_t^{-2}]$	watt m^{-2}	radiant pharosage	D_l	$[FL_t^{-2}]$	lumen m^{-2}	luminous pharosage
Q_r	$[PT]$	watt sec	radiant phos	Q_l	$[FT]$	lumen sec	luminous phos
U_r	$[PTL_t^{-2}]$	watt sec m^{-2}	radiant phosage	U_l	$[FTL_t^{-2}]$	lumen sec m^{-2}	luminous phosage
H_r	$[PL_t^{-4}L_r^2]$	herschel	radiant helios	H_l	$[FL_t^{-4}L_r^2]$	blondel	luminous helios
G_r	$[PL_t^{-4}L_r]$	herschel m^{-1}	radiant heliosent	G_l	$[FL_t^{-4}L_r]$	blondel m^{-1}	luminous heliosent
$J(\lambda)$	$[PL_t^{-2}\Lambda^{-1}]$	watt m^{-2} micron^{-1}	phengosage	—	—	—	—

TABLE II. Active concepts, IES system.*

Radiometric				Photometric			
Symbol	Dimensions	Unit	Name	Symbol	Dimensions	Unit	Name
Φ, P	$[P]$	watt	radiant flux	F	$[F]$	lumen	luminous flux
W	$[PL_t^{-2}]$	watt m^{-2}	radiant flux density	—	—	—	—
W	$[PL_t^{-2}]$	watt m^{-2}	radiancy (of source)	—	—	—	—
$H, \mathcal{E}$	$[PL_t^{-2}]$	watt m^{-2}	irradiancy (incident)	E	$[FL_t^{-2}]$	lux	illumination
U	$[PT]$	watt sec	radiant energy	Q	$[FT]$	lumen sec	quantity of light
u	$[PTL^{-3}]$	watt sec m^{-3}	radiant energy density	—	—	—	—
J	$[PL_t^{-2}L_r^2]$	watt sterad^{-1}	radiant intensity	I	$[FL_t^{-2}L_r^2]$	candle	luminous intensity
$N, \mathcal{B}$	$[PL_t^{-4}L_r^2]$	watt m^{-2} sterad^{-1}	steradiancy	B	$[FL_t^{-4}L_r^2]$	candle m^{-2}	brightness
—	—	—	—	B'	$[FL_t^{-4}L_r^2]$	lambert	brightness (lambert system)
U_λ	$[PT\Lambda^{-1}]$	watt sec micron^{-1}	spectral radiant energy	—	—	—	—
J_λ	$[PL_t^{-2}L_r^2\Lambda^{-1}]$	watt sterad^{-1} micron^{-1}	spectral radiant intensity	—	—	—	—

* *Illum. Eng.* **36**, 815 (1941).

TABLE III. Active concepts, OSA system.*

Radiometric				Photometric			
Symbol	Dimensions	Unit	Name	Symbol	Dimensions	Unit	Name
P	$[P]$	watt	radiant flux	F	$[F]$	lumen	luminous flux
W	$[PL_t^{-2}]$	watt m^{-2}	radiant emittance	L	$[FL_t^{-2}]$	lumen m^{-2}	luminous emittance
H	$[PL_t^{-2}]$	watt m^{-2}	irradiance	E	$[FL_t^{-2}]$	lumen m^{-2}	illuminance
U	$[PT]$	watt sec	radiant energy	Q	$[FT]$	lumen sec	luminous energy
u	$[PTL^{-3}]$	watt sec m^{-3}	radiant density	q	$[FTL^{-3}]$	lumen sec m^{-3}	luminous density
J	$[PL_t^{-2}L_r^2]$	watt sterad^{-1}	radiant intensity	I	$[FL_t^{-2}L_r^2]$	candle	luminous intensity
N	$[PL_t^{-4}L_r^2]$	watt m^{-2} sterad^{-1}	radiance	B	$[FL_t^{-4}L_r^2]$	candle m^{-2}	luminance
P_λ*	$[P\Lambda^{-1}]$	watt micron^{-1}	spectral radiant flux	—	—	—	—
H_λ**	$[PL_t^{-2}\Lambda^{-1}]$	watt m^{-2} micron^{-1}	spectral irradiance	—	—	—	—

* *J. Opt. Soc. Am.* **34**, 246, 255 (1944); **35**, 640 (1945).
** *J. Opt. Soc. Am.* **34**, 255 (1944).

pharosage *to* a surface, subscripts *i* and *e* may be used.[5]

Corresponding to this set of flux concepts is a set in which everything is multiplied by *time*. If power per unit area is such an important idea, one would expect that energy per unit area would be equally important; but no such concept appears in Table II, and no mention is made of watt second per square meter incident on a surface or watt second per square meter radiated from a surface.

[5] *i* for *incident* (Latin *incido*); *e* for latin *ex-* (=out of).

In the photometric columns of Table II, there is an inconsistently small number of concepts compared with the number in the radiometric column. One wonders why three separate concepts of *radiant flux density* should be necessary if the exactly similar requirements of the photometric system can be satisfied by one concept. On the other hand, two concepts of *brightness* are used on the right but only one on the left. The idea of *brightness* is undoubtedly the most confusing one in photometry, partly because of the use of two sets of units related by the factor π.

TABLE IV. Comparison of systems of nomenclature.
(a) Active radiometric concepts.

Dimensions	mks unit	IES (1941)		OSA (1944)		Proposed (1946)	
$[P]$	watt	Φ, P	radiant flux	P	radiant flux	F_r	radiant pharos
$[PL_t^{-2}]$	watt m^{-2}	W	radiant flux density	—	—	D_r	radiant pharosage
$[PL_t^{-2}]$	watt m^{-2} (radiated)	W	radiancy	W	radiant emittance	—	—
$[PL_t^{-2}]$	watt m^{-2} (incident)	H, $\mathcal{E}$	irradiancy	H	irradiance	—	—
$[PT]$	watt sec	U	radiant energy	U	radiant energy	Q_r	radiant phos
$[PTL_t^{-2}]$	watt sec m^{-2}	—	—	—	—	U_r	radiant phosage
$[PTL^{-3}]$	watt sec m^{-3}	u	radiant energy density	u	radiant density	—	—
$[PL_t^{-2}L_r^2]$	watt $sterad^{-1}$	J	radiant intensity	J	radiant intensity	—	—
$[PL_t^{-4}L_r^2]$	watt m^{-2} $sterad^{-1}$	N, $\mathcal{B}$	steradiancy	N	radiance	—	—
	herschel	—	—	—	—	H_r	radiant helios
$[PL_t^{-4}L_r]$	herschel m^{-1}	—	—	—	—	G_r	radiant heliosent
$[PL_t^{-2}\Lambda^{-1}]$	watt m^{-2} $micron^{-1}$	—	—	H_λ	spectral irradiance	$J(\lambda)$	phengosage
$[P\Lambda^{-1}]$	watt $micron^{-1}$	—	—	P_λ	spectral radiant flux	—	—
$[PT\Lambda^{-1}]$	watt sec $micron^{-1}$	U_λ	spectral radiant energy	—	—	—	—
$[PL_t^{-2}L_r^2\Lambda^{-1}]$	watt $sterad^{-1}$ $micron^{-1}$	J_λ	spectral radiant intensity	—	—	—	—

(b) Active photometric concepts.

Dimensions	mks unit	CIE (1938)		IES (1941)		OSA (1944)		Proposed (1946)	
$[F]$	lumen	F, φ	luminous flux	F	luminous flux	F	luminous flux	F_l	luminous pharos
$[FL_t^{-2}]$	lumen m^{-2}	—	surface density of luminous flux	—	—	—	—	D_l	luminous pharosage
$[FL_t^{-2}]$	lumen m^{-2} (radiated)	R	radiance	—	—	L	luminous emittance	—	—
$[FL_t^{-2}]$	lumen m^{-2} (incident)	E	illumination	E	illumination	E	illuminance	—	—
$[FT]$	lumen sec	L	quantity of light	Q	quantity of light	Q	"luminous energy"	Q_l	luminous phos
$[FTL_t^{-2}]$	lumen sec m^{-2}	—	exposure	—	—	—	—	U_l	luminous phosage
$[FTL^{-3}]$	lumen sec m^{-3}	—	—	—	—	q	luminous density	—	—
$[FL_t^{-2}L_r^2]$	lumen $sterad^{-1}$ [=candle]	I	luminous intensity	I	luminous intensity	I	luminous intensity	—	—
$[FL_t^{-4}L_r^2]$	candle m^{-2}	B	brightness	B	brightness (unrationalized)	B	luminance	—	—
	lambert, blondel	—	—	B'	brightness (rationalized)	—	—	H_l	luminous helios
$[FL_t^{-4}L_r]$	blondel m^{-1}	—	—	—	—	—	—	G_l	luminous heliosent

(c) Connective concepts.

Unit	CIE (1938)		IES (1941)		OSA (1944)		Proposed (1946)	
lumen $watt^{-1}$ (for homogeneous radiation)	K	luminosity factor of radiation	K	luminosity factor	K_λ	absolute spectral luminosity	$v(\lambda)$	spectral lamprosity
young $watt^{-1}$ (for homogeneous radiation)	K	relative luminosity factor	K	relative luminosity factor	$\bar{y}_\lambda$	relative spectral luminosity	$\bar{y}(\lambda)$	spectral lamprosity
lumen $watt^{-1}$ (for total radiation)	—	—	—	luminous "efficiency" of radiant energy	K	absolute luminosity	v_t	total lamprosity
young $watt^{-1}$ (for total radiation)	—	—	—	luminous "efficiency" of radiant energy	$\bar{y}$	relative luminosity	y_t	total lamprosity
lumen per radiated watt	—	"efficiency" of a luminous source	—	"efficiency" of a source of light	—	—	η	actance
Conversion factors								
lumen $young^{-1}$	—	—	—	—	K_m	maximum luminosity	c	conversion factor
young $lumen^{-1}$	—	"mechanical equivalent of light"	—	minimum "mechanical equivalent of light"	—	—	—	—

TABLE IV.—*Continued.*
(d) Passive properties of entities and materials.

Recommended Symbol	CIE (1938)		IES (1941)		OSA (1944)		Proposed (1946)	
$\rho(\lambda)$	ρ	spectral reflection factor	ρ	reflection factor	r_λ	spectral reflectance	$\rho(\lambda)$	spectral reflectance
$\tau(\lambda)$	τ	spectral transmission factor	τ	transmission factor	t_λ	spectral transmittance	$\tau(\lambda)$	spectral transmittance
ρ_r	—	—	—	—	r	radiant reflectance	ρ_r	radiant reflectance
τ_r	—	—	—	—	t	radiant transmittance	τ_r	radiant transmittance
α_r	—	—	—	—	A	radiant absorptance	α_r	radiant absorptance
ρ_l	ρ	reflection factor	ρ	reflection factor	R	luminous reflectance	ρ_l	luminous reflectance
τ_l	τ	transmission factor	τ	transmission factor	T	luminous transmittance	τ_l	luminous transmittance
α_l	α	absorption factor	α	absorption factor	—	—	α_l	luminous absorptance
κ	—	—	—	—	k	absorption coefficient	κ	afantivity
$\epsilon(\lambda)$	—	—	ϵ_λ	spectral emissivity	ϵ_λ	spectral emissivity	$\epsilon(\lambda)$	spectral stilbance
ϵ_t	—	—	ϵ_t	total emissivity	e	total emissivity	ϵ_t	total stilbance

(e) Colorimetric concepts.

Recommended symbol	CIE (1938)		IES (1941)		OSA (1944)		Proposed (1946)	
$\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$	$\bar{x}, \bar{y}, \bar{z}$	distribution coefficients of the equienergy spectrum	$\bar{x}, \bar{y}, \bar{z}$	stimuli	$\bar{x}, \bar{y}, \bar{z}$	color mixture data	$\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$	trichromatic weighting functions
X, Y, Z	—	—	X, Y, Z	tristimulus values	X, Y, Z	tristimulus values	X, Y, Z	trichromatic coordinates
(X, Y, Z)	—	—	—	color	—	color	(X, Y, Z)	trichros
x, y, z	x, y, z	trichromatic coefficients of the equienergy spectrum	x, y, z	trichromatic coefficients	x, y, z	trichromatic coefficients	x, y, z	homogeneous coordinates
(x, y)	—	—	—	chromaticity	—	chromaticity	(x, y)	chros
λ_d	—	hue wavelength	—	—	—	dominant wavelength	λ_d	dominant wavelength
λ_c	—	—	—	—	—	complementary wavelength	λ_c	complementary wavelength
p	—	purity or saturation	—	—	p	purity	p	akratos
T_c	—	color temperature	—	color temperature	—	color temperature	—	—

The usual "brightnesses" of Table II are unsatisfactory, not only because of the unnecessary complexity but also because they apply only to a surface source. In the recommended system of Table I, they have been replaced by *helios*, a sort of generalized brightness applying to any points in space, to any kind of medium, and to both surface and volume sources.[6] The concept of *heliosent* is required only in the calculation of radiation in absorbing, scattering, and radiating mediums. There is nothing in Table II corresponding to heliosent, since in defining active concepts the IES ignores all mediums that are not perfectly transparent.

As pointed out previously,[6] the idea of flux per unit solid angle (J and I of Table II) was important in the early stages of lighting but is of rapidly dwindling practical utility today. Thus there seems to be little value in having a separate concept for *intensity*. The concept does not appear in the recommended list of Table I; and if a case should arise in which flux per unit solid angle is needed, this ratio can be written as F/ω. No separate names and symbols are needed.

To summarize, the concepts of Table II are unsatisfactory because:

(*a*) The accent on flux per unit solid angle is antiquated and unnecessary; I and J should be omitted.

(*b*) There is no need for two or three flux densities. The single D of Table I suffices.

(*c*) The concept of brightness is confusingly complex, yet it lacks generality; it can be replaced by *helios*[7] with advantage.

(*d*) There is no need for both U_λ and J_λ. A more useful concept appears to be the $J(\lambda)$ of Table I.

[6] P. Moon, "A system of photometric concepts," *J. Opt. Soc. Am.* **32**, 348 (1942); "The names of physical concepts," *Am. J. Physics* **10**, 134 (1942).

[7] P. Moon and D. E. Spencer, "Helios and brightness," *Illum. Eng.* **39**, 507 (1944).

It is easy to see how the inconsistencies of Table II developed. The IES has been interested almost exclusively in the photometric concepts rather than the radiometric. Furthermore, attention has been riveted on room lighting, where distances are ordinarily so small that atmospheric absorption and scattering can be neglected. Thus it was natural to confine the concepts to the practical necessities of interior lighting, producing narrow definitions that do not satisfy the needs of the physicist, the astrophysicist, or the meteorologist. It is desirable to replace the narrow concept of "brightness of a surface" by the general concept of *helios* at any point and to introduce the new concept of *heliosent*.[8]

Until recently there was no radiometric system in the IES definitions. Not until 1941 was the system of Table II incorporated into the "Illuminating Engineering Nomenclature and Photometric Standards." Obviously, the IES radiometric set of concepts is not consistent with the photometric set. The former has received little use and is not taken seriously by most IES members.

The OSA system of Table III presents a carefully considered parallelism between the radiometric and the photometric concepts. An exact correspondence is found between the surface densities W, H and L, E, and between the quantities U, u and Q, q. Nevertheless, the OSA system has all the defects (*a*), (*b*), (*c*), (*d*) mentioned in connection with the IES system.

Perhaps the worst offender against internationality is the ending -ANCY used by the IES in their words "radiancy," "irradiancy," and "steradiancy." The Commission Internationale de l'Eclairage (CIE) does not employ this ending, neither does the OSA, and no example of its use could be found in other branches of engineering. The suffixes -ANCY and -ANCE come from the same Latin ending -ANTIA. The particularly reprehensible aspect of -ANCY is that there seems to be no way of distinguishing it from -ANCE except in English. In Dutch, for instance, -ANCE is translated as -ANTIE, which sounds almost exactly like English -ANCY. In French, German, and other languages there is likewise no known way of representing this slight difference in ending which is supposed to indicate a big difference in meaning. Furthermore, the ending is applied to some kind of an active concept, which is at variance[2] with international words making use of the Latin -ANTIA.

2. Names and Symbols

Having considered the concepts themselves, we can now turn to the names of these concepts and the symbols used to designate them. The customary method of selecting the symbols gives no indication of the parallelism existing between the radiometric and the photometric concepts. In Table II, there is nothing to suggest that Φ and F are closely related, or H and E, or N and B. It is simply a question of memorizing the 12 symbols. Similarly, in the OSA system (Table III) 14 distinct symbols must be memorized. A great simplification is effected by employing the same symbol for corresponding radiometric and photometric quantities. In Table I the number of distinct symbols is reduced to seven. If a discussion deals with only one system, as is usually the case, subscripts are unnecessary; but if there is any chance of confusion, subscripts r and l are employed to distinguish the radiometric and photometric quantities.

Further simplification can be obtained by using the same name for corresponding concepts, as shown in Table I. In the IES nomenclature, it is not obvious from the words that the concepts *illumination* and *irradiancy* correspond, or that *quantity of light* goes with *radiant energy*, or *brightness* with *steradiancy*. The OSA system represents a sincere and careful analysis of the difficulties of the older system and an attempt to remedy them. The 11 distinct names of Table II (not counting the adjectives "radiant" and "luminous") are cut down to 9 distinct names in Table III. The proposed international system (Table I) employs only seven.

3. Detailed Criticism

A comparison of the names of concepts is facilitated by use of Table IV. This table includes the names used by the Commission Internationale de l'Éclairage (CIE),[9] the Illuminating Engi-

[8] See *An unofficial guide to photometric nomenclature* (to be published).

[9] *International lighting vocabulary* (Commission Internationale de l'Éclairage, Teddington, England, 1938).

neering Society (IES),[3] the Colorimetry Committee of the Optical Society of America (OSA),[4] and the system proposed in this paper. The OSA and the proposed systems will be considered especially, because they are the most recent. In the OSA system, one notes the following:

(*a*) The ending -ANCE is used for active concepts called emittANCE, illuminANCE, luminANCE, irradiANCE, radiANCE. But it was shown previously[2] that this international ending should be reserved for *passive* concepts to denote a property of an entity.

(*b*) The ending -ITY is used by the OSA for the active concepts densITY and intensITY. It is also employed by IES and OSA in emissivITY, purITY, and chromaticITY. The interests of international standardization would be furthered by reserving this ending for the specific meaning of a passive property of a material.

(*c*) If the word *illuminance* were accepted for luminous flux incident on a surface per unit area, then one might logically expect that *luminance* would refer to the most closely related concept, namely, the luminous flux *from* a surface per unit area. But it refers to another concept having different dimensions.

(*d*) The words *luminance* and *illuminance* are so nearly alike that they are almost indistinguishable in spoken English. This objection should be enough to eliminate them, even without the objections of noninternationality and use of an ending that should be reserved for something else.

(*e*) For continuous spectra, it is necessary to have a new concept expressing quantity per unit wavelength band. In the OSA system, such a concept is called *spectral radiant flux* P_λ (watt micron^{-1}) to show that it is related to *radiant flux* P (watt). But it is inconsistent to call these two concepts by the same name when they have different dimensions, while using distinguishing terminology for the other concepts. If *radiant flux* P (watt) and *spectral radiant flux* P_λ (watt micron^{-1}) have the same symbol, why not talk of *areal radiant flux* P_A (watt m^{-2}) in place of *irradiance* and *angular radiant flux* P_ω (watt steradian^{-1}) in place of *radiant intensity?*

Actually, it seems preferable to designate all of these quantities by different names and symbols. Note also the inconsistency of employing a subscript λ to change P to the entirely different quantity P_λ, and using the same device to change reflectance r to reflectance r_λ, both quantities having the same dimensions.

(*f*) The word *density* is often employed in physics to indicate a quantity per unit area. *Luminous density* might very well be expressed in lumens per unit area. Yet in the OSA system it is expressed in lumen seconds per unit volume. In any case, such words as *density*, *intensity*, *quantity* and *strength* as part of the name of a scientific concept produce clumsy expressions whose use should be discouraged. The lack of internationality of the ending -ITY used in this sense was mentioned in (*b*).

(*g*) The quantity Q, expressed in lumen seconds, is called "luminous energy" in the OSA system. But it is not energy and should not be called energy. Energy is expressed in watt seconds in the *mks* system: it cannot possibly be expressed in lumen seconds. The claim that the adjective "luminous" makes the energy into something that is not energy is like talking about "cold heat."

(*h*) At least three symbols have been employed at various times for the function that relates the radiometric and photometric quantities. Obviously, only one of the symbols v, K and $\bar{y}$ is needed. Since $\bar{y}$ is thoroughly international in colorimetry,[10] the logical procedure would be to use it also in photometry. Unfortunately, such a step does not seem to be feasible at the present time. We use the symbol $v(\lambda)$ in the photometric system and $\bar{y}(\lambda)$ in the colorimetric. In any case, K should not be introduced. The symbol is employed universally to indicate a *constant*, not a continuous function of wavelength.

(*i*) The function $v(\lambda)$ is called *luminosity factor* by the CIE and IES and *luminosity* by the OSA. Obviously, $v(\lambda)$ is not a "factor" but a continuous function of λ. The word *luminosity* is also unfortunate because it has several other meanings, which may cause ambiguity, and because it is not international (English *luminosity*, Fr. *visibilité*, It. *visibilità*, G. *spektrale Hellempfindlichkeit*, R. *svetlota*). The use of a new

[10] Commission Internationale de l'Eclairage, *Comptes rendus des séances* (London, 1931), p. 25.

TABLE V. Simplified sets of concepts for ordinary use.

Radiometric			Photometric		
Symbol	Name	Unit	Symbol	Name	Unit
		IES system			
Φ	radiant flux	watt	F	luminous flux	lumen
H	irradiancy	watt m^{-2}	E	illumination	lumen m^{-2}
J	radiant intensity	watt $sterad^{-1}$	I	luminous intensity	candle
N	steradiancy	watt m^{-2} $sterad^{-1}$	B	brightness	candle m^{-2}
		OSA system			
P	radiant flux	watt	F	luminous flux	lumen
H	irradiance	watt m^{-2}	E	illuminance	lumen m^{-2}
J	radiant intensity	watt $sterad^{-1}$	I	luminous intensity	candle
N	radiance	watt m^{-2} $sterad^{-1}$	B	luminance	candle m^{-2}
		Proposed international system			
F_r	radiant pharos	watt	F_l	luminous pharos	lumen
D_r	radiant pharosage	watt m^{-2}	D_l	luminous pharosage	lumen m^{-2}
H_r	radiant helios	herschel	H_l	luminous helios	blondel

word is needed,[11] such as *lamprosity*, which can be assimilated by all European languages.

(*j*) The quantity $v(\lambda)$ is expressed in lumens per watt. Another quantity having the same dimensions and expressed in the same unit is what is generally called "luminous efficiency" or *luminous efficacy*. It is exactly the same as $v(\lambda)$ except that it applies to a complete spectrum instead of a homogeneous radiation. Thus it should not be given a separate name and symbol. In the proposed international system, it is called *total lamprosity* v_l.

(*k*) The usual term *emissivity of a radiator* may be criticised because

(1) The ending is wrong. The value depends on both the material and how it is arranged. In the form of a complete enclosure at uniform temperature, *any* material gives $\epsilon = 1.00$. Thus the international ending must be -ANCE, not -ITY.

(2) The concept has no connection with thermionic emission. It deals with *radiation*, and thus the use of the word *emissivity* is likely to cause confusion. The word chosen in the proposed international system is *stilbance*.

(*l*) The quality of a color is often called *chromaticity*. This name violates the principle of internationality: it does not designate a property of a material but is used for what is actually an -OS concept. Our suggestion is *chros*.

(*m*) Another term employed in colorimetry is *purity*. As with *chromaticity*, the -ITY ending is incorrect. We suggest *akratos*.

[11] P. Moon, "Basic principles in illumination calculations," *J. Opt. Soc. Am.* **29**, 108 (1939).

4. Simplified Sets

The preceding sections have pointed out some of the defects of the IES and OSA systems of nomenclature, with particular accent on internationality. These defects are believed to be eliminated by the proposed international system. On the other hand, the new system requires that a number of very unfamiliar words be memorized. In practice, however, this does not appear to constitute a real stumbling block. Five year's experience in teaching the system to students has failed to show any difficulty. On the contrary, the nomenclature is learned more easily than the IES nomenclature and seems to result in clearer thinking and less confusion.

For the average illuminating engineer, the formidable array of new words is unnecessary since he employs only a few concepts in his daily work. The concepts most commonly used are shown in Table V. In the IES photometric system, only four concepts are necessary in handling the great bulk of practical work. Four are required also in the OSA system. In the proposed international system, the number is reduced to three because luminous intensity has been abandoned. Table VI indicates that the number of unfamiliar words that must be learned in the new system is actually less than in the

OSA system, while the number of symbols that must be kept in mind is greatly reduced over either the OSA or the IES system.

5. Internationality Again

Obviously, one of the foremost considerations of any nomenclature committee must be the behavior of its proposed names in other languages. But, strange to say, no inventor of a new term seems ever to let his thoughts roam beyond the boundaries of his own country. In view of the global nature of science, this procedure is almost crazy enough to be worthy of a politician.

The reader may wonder why we insist that our proposed words are international while those of the OSA are not. The answer lies in the behavior of scientific names that have been introduced during several centuries. In most cases, a common word has been taken from the dictionary and given an additional and highly specialized meaning. History shows that such a word is not international, for it is translated into each language in which it is used.[2] Since a one-to-one correspondence does not exist between words in different languages, various scientists translate the same term in different ways. This leads to confusion. To eliminate this confusion a national or international board must next decide which of the various translations is to be official. This official word must then be memorized.

Contrast the foregoing with the behavior of unfamiliar words taken from classical Greek. The new word cannot be translated since it has no obvious equivalent in popular speech. Thus it is accepted with little or no change by all languages. The word is not English, or French or Russian, so no national pride is injured by accepting it. An example is *energy*, a Greek word that is written *énergie* in French, *energia* in Italian, *energia* in Spanish, *Energie* in German, *energie* in Dutch, and *energīa* in Russian. On the other hand, the Latin-derived *force* is carefully translated and becomes the *Kraft* of German, the *kracht* of Dutch, the *sīla* of Russian.

That the usual procedure has resulted in a noninternational photometric terminology is obvious from Table I of a previous paper.[1] The mess is even worse than one might logically expect, since, although it would seem reasonable for English and all the Romance languages to employ the same terms, this is seldom true. The same noninternationality would surely occur with the proposed OSA words, for they have Latin roots and are closely associated with many English words. Also, as shown previously, the OSA terms employ word endings in a noninternational manner.

6. Summary

The paper has outlined a method of naming scientific concepts to achieve (*a*) nonambiguity, and (*b*) internationality. The method consists in selecting ancient Greek words and employing a set of international word endings.

To show that this procedure results in a workable system of names, we have applied it to the concepts of radiometry, photometry and colorimetry. The resulting system is compared with three other systems which are in use or have been proposed. It is found that the suggested international system of concepts and names is simpler and more logical than the other systems. It is free from the ambiguity and danger of confusion that are always present when words in common speech are forced to serve also as scientific designations. It is international, thus eliminating much of the difficulty of reading scientific literature in foreign languages. It even compares favorably with the older nomenclature with respect to the number of symbols and the number of new words that must be learned.

One may argue that, though it is all very pretty to play with logical sets of names and definitions, the present names are so firmly established that they will continue to be used in practice. The fact is, however, that terms dealing with light are by no means as firmly standardized as one might expect. The only ones that have

TABLE VI. A comparison of the three simplified sets of Table V.

System	No. of concepts	No. of symbols	No. of unfamiliar names
IES	8	8	2
OSA	8	8	4
New	6	3	3

received any considerable support are those on the right-hand side of Table II. These photometric concepts have had wide acceptance and may continue to exist until growing technological needs indicate their inadequacy. Very little standardization exists in radiometric nomenclature. The new names of Table III have been employed to some extent in the United States, but principally by members of the OSA Colorimetry Committee. Thus it is not impossible that modifications may be made in existing nomenclature. And whether official sanction is given or not, there are still some unregimented scientists who will use a new word if it suits them.

In conclusion, it is a pleasure to thank those who helped to clear up difficulties (linguistic or otherwise) which were encountered in writing this paper. We are deeply grateful to Elliot Q. Adams, H. E. Blair, Erich Berger, Salvador Castro, G. Alan Connor, E. C. Crittenden, F. Cuétara, F. M. Currier, Oswaldo P. Doria, Roberto M. Fano, I. H. Godlove, J. P. Den Hartog, International Auxiliary Language Association, H. Jacob, Deane B. Judd, Waldemar Kaempffert, R. F. Koch, W. M. Locke, D. L. MacAdam, Sanford A. Moss, W. B. Nottingham, Mario A. Pei, Duane Roller, Pieter Schoonhoven, Lewis L. Sell, Dirk J. Struik, Gregory Timoshenko, and George A. Znamensky.

RECENT MEETINGS

Illinois Section

The Illinois Section of the American Association of Physics Teachers held its fall meeting on November 23, 1946 at the University of Illinois. The following program was presented.

Two useful laboratory devices: a falling-body release and a specific-resistance frame. C. R. Smith, *Aurora College.*

Induced longitudinal vibrations in a metal rod by solid carbon dioxide and a flame. C. Howard, *Wheaton College.*

A photoelectric liquid-level controller. C. Ireland, *Staley Manufacturing Company.*

Phase change of longitudinal waves on reflection. D. L. Eaton, *Northern Illinois State Teachers College.*

Crystal rectifiers. W. E. Meyerhof, *University of Illinois.*

A method of determining the index of refraction of liquids. G. Lefler, *Eastern Illinois State Teachers College.*

The electronic switch in experiments. O. L. Railsback, *Eastern Illinois State Teachers College.*

Teaching physics for the Army in Florence, Italy. W. W. Mutch, *Knox College.*

Development of the betatron for electron therapy. L. S. Skaggs, *University of Illinois.*

At the business meeting officers for 1947 were elected. Following the program there was an inspection of the research facilities in physics at the University of Illinois.

W. H. Ellers,
President

Oregon Section

The forty-third meeting of the Oregon Section of the American Association of Physics Teachers was held on November 9, 1946 at the University of Oregon. The following program was presented.

A new course in radiation physics. E. G. Ebbighausen, *University of Oregon.*

A report on the Cornell conference on luminescent materials. R. T. Ellickson, *Reed College.*

A new course in electricity. M. A. Starr, *University of Oregon.*

Power-measuring devices for the very high radio frequencies. E. A. Yunker, *Oregon State College.*

Airborne radar. W. P. Dyke, *Linfield College.*

Development of concept and the teaching of science. A. V. Shatzel, *Lewis and Clark College.*

Plans for a new classroom-laboratory building. W. V. Norris, *University of Oregon.*

It was pointed out at the meeting that the International Committee of Weights and Measures in its 1946 meeting will probably adopt one of the meter-kilogram-second systems of units for electric quantities, to replace the International system in use heretofore. This action would be in accordance with the decision made in 1935, which was to have gone into effect in 1940 but was upset by the war. The new system will presumably be officially instituted on January 1, 1948. It was suggested by H. B. Cockerline and W. R. Varner that when the new units are adopted a standard system of symbols and of names should also be chosen, and that physicists should have as much interest in the selection as electrical engineers have had in the past. They presented a set of proposed symbols and names for units in the cgses, cgsem, mks and MKS system, intended to bring about consistency in the names for the same quantity in these four systems. One new name is suggested—the ***kenn***; this is the unit of electric flux, named for A. E. Kennelly.

Wm. R. Varner,
Secretary

NOTES AND DISCUSSION

A Model for the Demonstration of Steiner's Theorem

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STEINER'S theorem for moments of inertia will become more clearly visualized and will have more physical meaning if it is demonstrated experimentally. W. V. Burg[1] recently described a laboratory experiment on the theorem. The apparatus shown in Fig. 1 is intended as a lecture demonstration of the same principle; it was designed by Professor G. W. Stewart.

The device consists of a bicycle wheel mounted in a frame of light steel tubing to which it may be locked by a setscrew a in the crossbar. The distance from the axle b to the knife-edges c is equal to the radius of the wheel. The rim is weighted by a heavy lead band.

If the knife-edges are placed on a horizontal bearing surface and the device is set swinging as a pendulum, the period is found to be longer with the wheel locked than with it free. In the latter case, the wheel rotates only about the knife-edges; so the moment of inertia is ml^2, where m is the mass of the wheel, and l is its radius. When the wheel is locked, however, it is forced to rotate also about the hub; so the moment of inertia is ml^2+I_c, where I_c is the moment of inertia of the wheel about its center of mass.

The two periods of oscillation have the ratio,

$$\frac{T_{\text{lock}}}{T_{\text{free}}}=\frac{2\pi(2ml^2/mgh)^{\frac{1}{2}}}{2\pi(ml^2/mgh)^{\frac{1}{2}}}=2^{\frac{1}{2}}.$$

The results obtained with this apparatus are good enough for a lecture demonstration, the ratio of the periods being approximately 1.38 for the model described.

An interesting extension of the experiment is to give the wheel an added constant spin before setting the device into vibration. The rotation about the hub owing to the pendular motion is then alternately opposing and reinforcing the spin velocity; so the wheel is accelerated positively when swinging one way and negatively when moving the other. For the greatest effect, the spin should equal the maximum velocity of the alternating rotation.

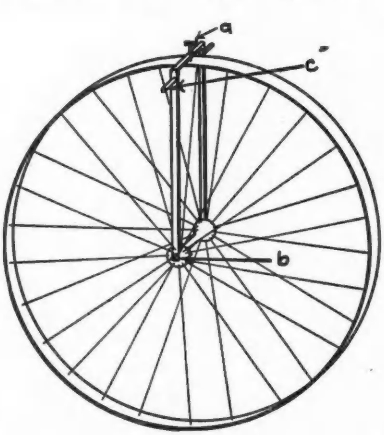

FIG. 1. Apparatus for demonstrating Steiner's theorem.

[1] W. V. Burg, "An experiment on Steiner's theorem," *Am. J. Physics*, 14, 196 (1946).

New Method for Demonstrating the Addition of Two Isochronous and Perpendicular Vibratory Motions

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THE experiment described in this note is believed to be new. The apparatus is simple, and the experiment is easily understood by students who are beginning the study of the addition of harmonic vibrations.

In Fig. 1, A_1M_1 and A_2M_2 are two simple pendulums of the same length. A light, rigid rod B_1BB_2 is fastened at B_1 to the string A_1M_1, and passes through a small ring fastened at B_2 to the string A_2M_2. The pendulum BM is supported halfway between B_1 and B_2. Its length is the same as that of either of the other two pendulums. For a successful experiment, the masses M_1 and M_2 should be equal, and the mass M about one-hundredth that of M_1 or M_2.

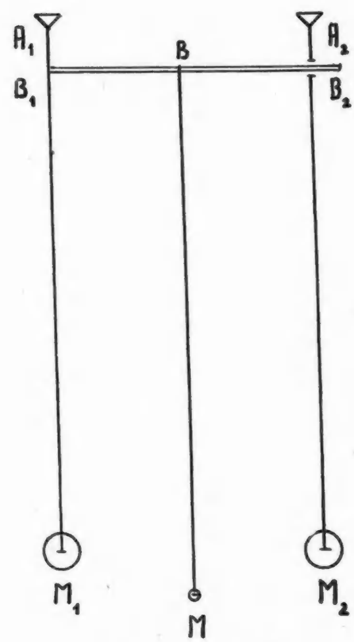

FIG. 1. Diagram of apparatus.

Good results were obtained with pendulums 40 cm in length, and with masses of 100 and 2 gm, respectively. The position of the rod should be such that $A_1B_1/A_1M_1=0.06$; A_1A_2 is 20 cm.

(*a*) When M_2 is at rest and M_1 is oscillating in the plane of the figure, pendulum BM will accompany, by resonance, the oscillations of A_1M_1 in the same plane as that of the figure.

(*b*) When M_1 is at rest and M_2 is oscillating in a plane perpendicular to that of the figure, M will oscillate, by resonance, in a plane also perpendicular to that of the figure.

(*c*) When M_1 oscillates in the plane of the figure and M_2 simultaneously oscillates in a plane perpendicular to that of the figure, the motion of M will be the resultant of the vibratory motions of M_1 and M_2. The following cases occur:

(1) When M_1 and M_2 are in phase, M describes approximately an arc with center at B.

(2) When M_1 and M_2 are in opposition (phase angle π), M describes an arc with center at B, in a plane perpendicular to that of the preceding case.

(3) When M_1 and M_2 differ in phase by $\frac{1}{2}\pi$, M describes a circle.

(4) When M_1 and M_2 differ in phase by $\frac{3}{2}\pi$, M describes a circle in the opposite sense.

(5) In the intermediate cases, ellipses will be obtained.

It would be interesting to make a mathematical study of the resulting motions when the three pendulums are not isochronous.

Boners

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OUT-OF-SCHOOL behavior—or lack of it—proves most physics teachers loyal adherents to the belief that "a little nonsense now and then is relished by the wisest men." Classroom propriety allows few if any to express by increased term grades their appreciations for the *relish* which the students—not infrequently the worst—gratuitously supply.

It is therefore suggested that a small section of the *Journal* occasionally be devoted to honoring the anonymous numbskull, and to recording for the enjoyment of others his worthy contributions to physics and mathematics. By admitting certain submitted boners to the *Journal*, the Editor can *re*stimulate the teacher as he labors over his stack of papers to be graded. Perhaps the paper which formerly would have been tossed aside as worthless will now be more carefully perused, and eventually find its true place in the literature.

In Dr. Seuss' books of boners,[1] said to be assembled from teachers' contributions of their pupils' endeavors, the editor *may* have invented definitions such as

dyne—*a thin coin equal to two nickels.*
tube of force—*a lumber term used especially by carpenters.*

Yet misunderstanding of terminology gibraltarized in physics does form a natural source of excellent boners—and simultaneously a clear confession that the textbook was never opened. Witness two cases from the 1946 final examination in freshman physics at George Washington University:

Moment of inertia is defined as *the semination of the mass by the square of the radius;*

Power of a lens is *the invert of the focal length measured in meters, and is expressed in diapers.*

Only authentic boners, fruits of student labor on problems or examinations, are likely to be of interest. If the student marks as correct on a multiple-choice or true-false test some obviously ridiculous answer, the boner is the examiner's and should be duly accredited to him.

[1] Alexander Abingdon (Dr. Seuss), *Boners, more boners, still more boners* (Blue Ribbon Books, 1934).

Completely Inverted Images

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SEVERAL months ago a note[1] with the title above was contributed to these columns. It pointed out that sand grains on a rough plastered wall look like indentations when observed through a galvanometer telescope, and it stated that many observers would be persuaded by this demonstration that simple telescope images are inverted in the axial direction.

It was a surprise to learn that one reader[2] had taken the proposed explanation seriously and had gone to the trouble of demolishing it in public. I hasten to say that my note on axial image inversion was intended as a puzzle and not as a revolutionary discovery in optics.

[1] P. Kirkpatrick, *Am. J. Physics* **13**, 203 (1945).
[2] G. L. Walls, *J. Opt. Soc. Am.* **36**, 615 (1946).